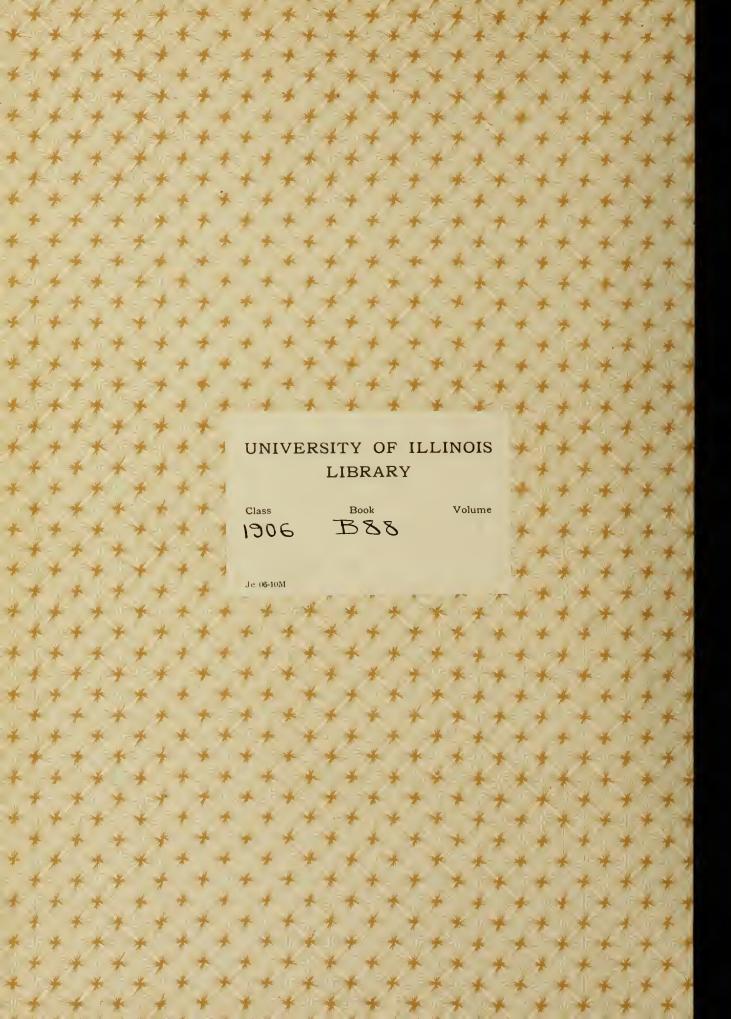
BUMANN.

Design for a
Three-Hinged Reinforced
Concrete Arch

Civil Engineering

BS

1906







DESIGN



FOR A

THREE-HINGED REINFORCED CONCRETE ARCH

 \mathbf{BY}

CECIL SPENCER BUMANN

THESIS

FOR

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

PRESENTED JUNE 1906



UNIVERSITY OF ILLINOIS

May 26, 1906

This is to certify that the thesis prepared under the immediate direction of Instructor L. A. Waterbury by

CECIL SPENCER BUMANN

entitled DESIGN FOR A THREE-HINGED REINFORCED-CONCRETE ARCH is approved by me as fulfilling this part of the requirements for the Degree of Bachelor of Science in Civil Engineering.

Fra O. Baker.

Head of Department of Civil Engineering

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DESIGN FOR A THREE-HINGED REINFORCED CONCRETE ARCH.

The object of this thesis was to design a three-hinged reinforced concrete arch bridge which would be suitable for some particular place. The site which was selected is about one-half mile north of Mahomet, Illinois, at a place where the Sangamon River is crossed by a public highway. At this point there is at present an iron bridge of the bow string type having one 100-foot span and two 75-foot spans.

DETERMINATION OF WATER WAY.

It was first necessary to determine the water way required. To do this inquiry was made concerning the height of high water under the present bridge. The water level of the highest water was said to have been about two feet below the floor of the old bridge. This level was marked on a cross-section of the stream at the bridge site and the water area thus obtained was determined by measuring the ordinates included between the high water line and the ground line. The area thus obtained was 3880 sq. ft. To check this result the drainage area above the bridge was estimated from county and state maps and the water way required was computed by Talbot's formula which is,



,

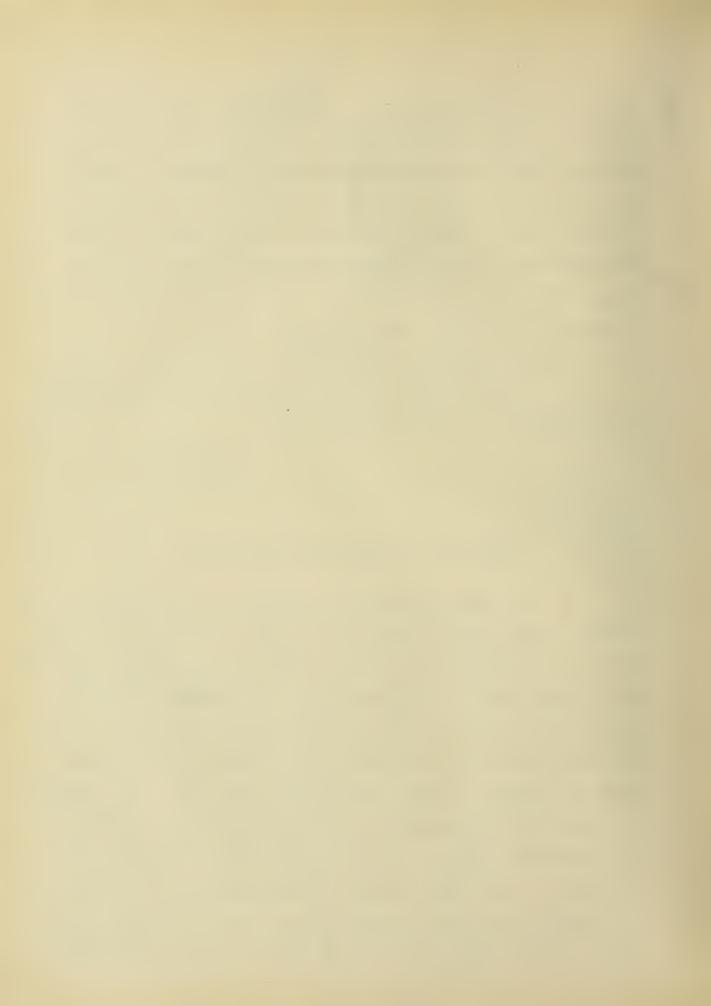
Area of water way in sq. ft. = $c \sqrt{(Drainage area in acres)^3}$ in which c is a constant depending on the character of the basin drained. "For rolling agricultural country subject to floods at times of melting snow, and with the length of the valley three or four times its width, c is about 1/3." (See Baker's Masonry Construction, p. 395.) As this describes the area above the bridge site c was chosen as 1/3. The drainage area was found to be about 360 sq. miles = 230,400 acres. Then,

Water way = 1/3 $\sqrt{(230,400)^3}$ = 3500 sq. ft.

From the above result it was concluded that a water way of 3800 sq. ft. would be sufficient.

SELECTION OF DIMENSIONS FOR BRIDGE.

The next step was the selection of dimensions for the bridge. It was first thought best to have the elevation of the floor of the new bridge the same as that of the present one. With this in mind arches of 75-foot span, the intrados of each of which was an arc of a circle, were drawn on the profile of the cross-section of the stream and it was found that four arches would be required to give the necessary water way. This design was unsatisfactory because it would have required too long and too expensive a bridge. By raising the floor elevation seven feet and by using three arches of 82-foot span and 11.8-foot rise the necessary water way was secured and the total length of the bridge required would be nearly the same as that of the present



structure. These dimensions were adopted for the design.

PRELIMINARY DESIGNS.

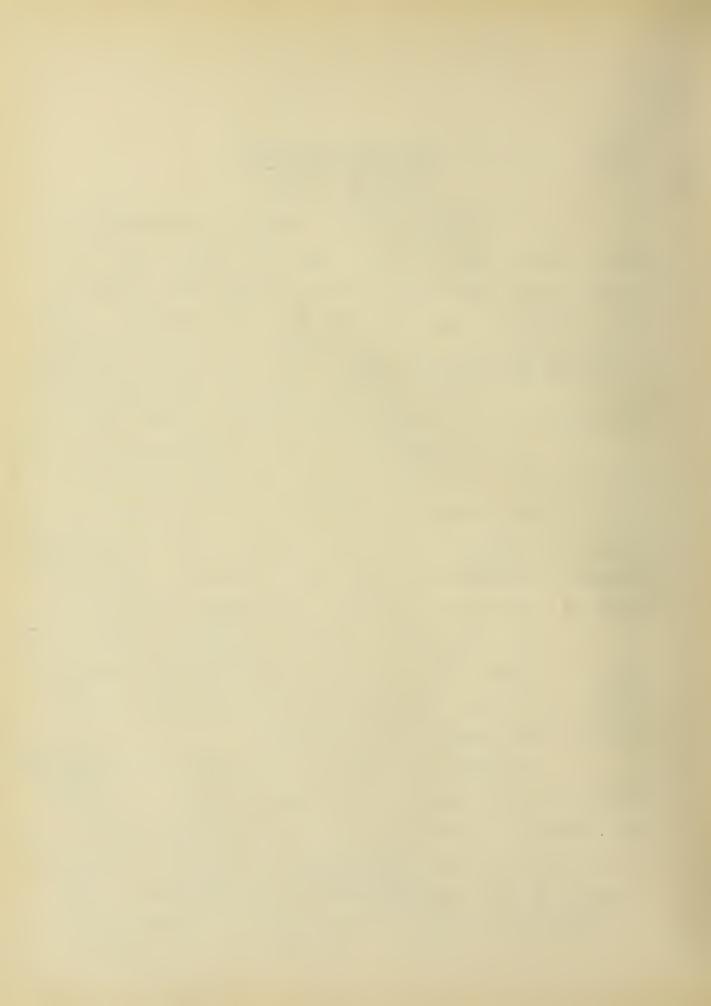
FIRST TRIAL ARCH.

The next step was the design of a three-hinged reinforced concrete arch of 82-foot span and 11.8-foot rise. A circular arch was first assumed having a radial depth of one foot six inches at the crown and three feet at the skewbacks. These dimensions are about the same as those of a reinforced concrete at Grand Rapids, Mich. ft. arch, which has a span of 83, and a rise of 11 feet 7 inches, the description of which is given in the Michigan Engineer for 1905.

A longitudinal section of the proposed arch was drawn to a scale of 1 inch = 4 feet, and the half arch and superincumbent earth was divided into 7 sections as shown in Plate I. The weight of earth and concrete for a section one foot in width was computed from dimensions scaled from the drawing. Earth was assumed to weigh 100 lb. per cu. ft. and concrete 150 lb. per cu. ft. Since the arch is three hinged there will be no bending moment at the center of the arch nor at the skewbacks, so the horizontal and vertical components of the reactions can be determined by taking moments about the hinges.

Assuming the thickness of the earth filling above the crown to be six inches, the total weight of earth and concrete for one-half of the arch was found to be 30,630 lb. which is equal v_d to the vertical component of the dead load reaction at the skewbacks.

The center of gravity section was assumed to be at the middle of the section. While this is not strictly true it



is close enough for the preliminary examination. The distance of these centres of gravity from the vertical line through the hinge at the abutment is given in the column marked X in Table I.

Let W = the total weight of earth and concrete in a section

Wx = the moment of W about the skewback hinge

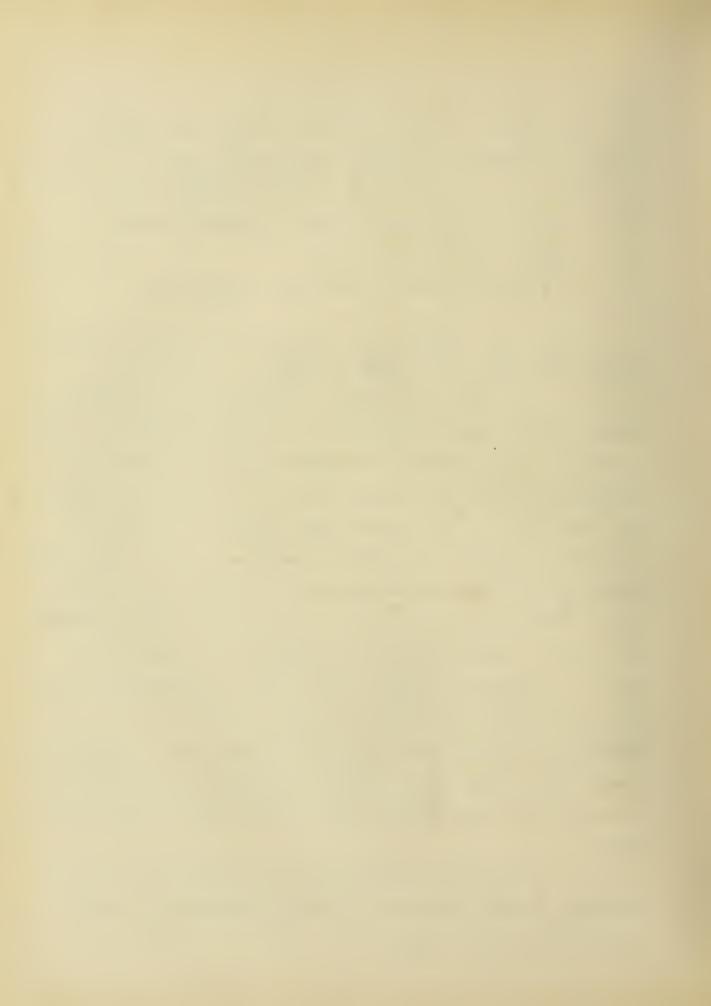
€Wx = sum of these moments

 H_d = the horizontal thrust due to dead load

Then, $H_d = \frac{\xi \text{Wx}}{11.8}$ since 11.8 feet is the rise of the arch. So, for this arch, $H_d = \frac{\xi \text{Wx}}{11.8} = \frac{446100}{11.8} = 37800 \text{ lb.}$

The line of pressure of these forces was drawn by laying out the funicular polygon for the loads obtained. (If the funicular polygon is started through one hinge it should pass through the other, thus giving a check on the work). (See Plate I.) The center line of the arch was next shifted to coincide with the line of pressure, as nearly as possible, so that there would be no bending moment in the arch due to the dead load. The required radial depth at a number of sections was then determined as follows. The live load was assumed as 100 lb. per sq. ft. or 6000 lb. concentrated load, 6000 lb. being the concentrated load which will produce about the same bending moment in any one foot section as a fifteen-ton steam roller having axles 11 feet apart with 6 tons on the forward roller, four feet wide and 4 1/2 tons on each of two rear rollers 5 feet between centers and 20 inches wide.

The center of gravity of the uniform load over the half arch is 20.5 feet from the skewback hinge so the horizontal thrust due to this loading,



$$H_{u.1.} = \frac{4100 \times 20.5}{11.8} = 7100 \text{ lb.}$$

And the vertical component of the reaction, $V_{u.1.} = 4100 \text{ lb}$. Adding these to the corresponding values for the dead load,

$$H = H_d + H_{u.1.} = 37800 + 7100 = 44900 lb.$$

$$V = V_d + V_{u.1} = 30630 + 4100 = 34730 lb.$$

$$R = \sqrt{H^2 + V^2} = \sqrt{(44900)^2 + (34730)^2} = 56800 \text{ lb.}$$

in which R is the resultant at the skewback.

Taking the concentrated load at the crown, in which position it will produce the greatest thrust,

$$V_{c.1.} = 3000 \text{ lb.}$$

$$H_{c.1.} = \frac{42V}{11.8} = \frac{42 \times 3000}{11.8} = 10680 \text{ lb.}$$

$$H = H_d + H_{c.1} = 37800 + 10680 = 48480 lb.$$

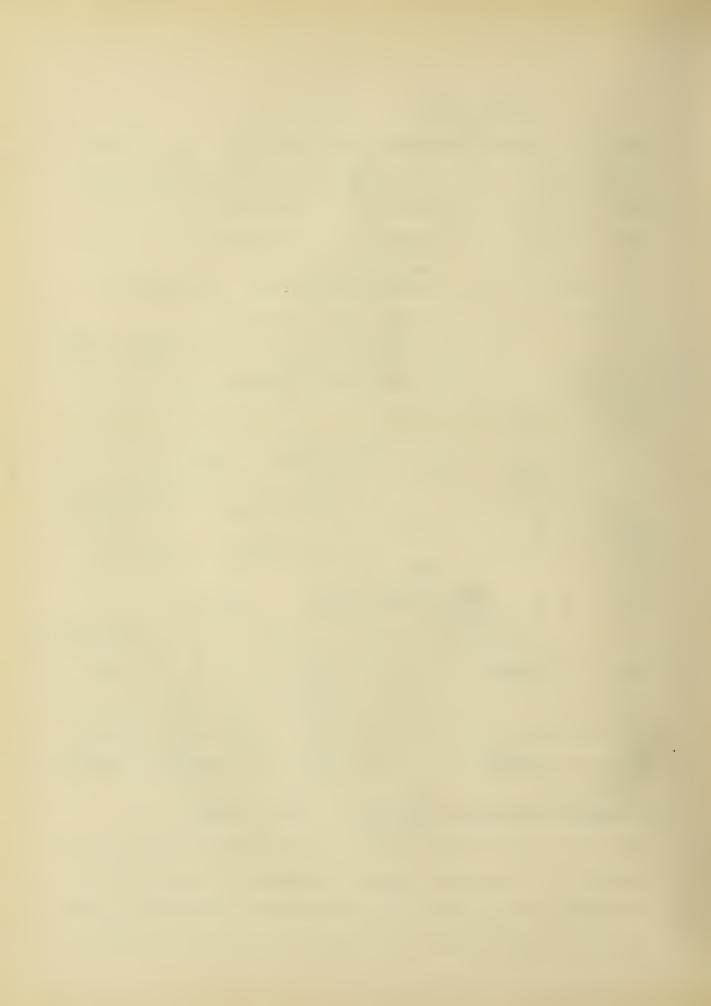
$$V = V_d + V_{c.1} = 30630 + 3000 = 33630 lb.$$

$$R = \sqrt{(48480)^2 + (33630)^2} = 59000 \text{ lb.}$$

The maximum pressure which most bridge specifications allow on masonry under plates is 250 lb. per sq. in. This limit was used for the pressure of the hinge plates upon the concrete. The thickness required at the crown to fulfil this condition would be $\frac{48480}{12 \times 250} = 16.15$ inches. The required thickness at

the skewbacks would be
$$\frac{59000}{12 \times 250} = 19.7$$
 inches.

This arch was therefore heavier than required. In reducing the weight of the arch the required thickness is also reduced so a thickness of 14 inches at the crown and 18 inches at the skewbacks was next assumed. The thickness required at the point of



maximum moment was then determined. Since the design intends to practically eliminate dead load moment the moment considered was that due to the concentrated load of 6000 lb.

Calling the bending moment which produces compression at the extrados of the arch positive and that which produces tension negative, the position of the load which produces the maximum positive moment will be nearly at the quarter points and the maximum moment will be under the load. For the live load in that position,

V = 4500 lb.

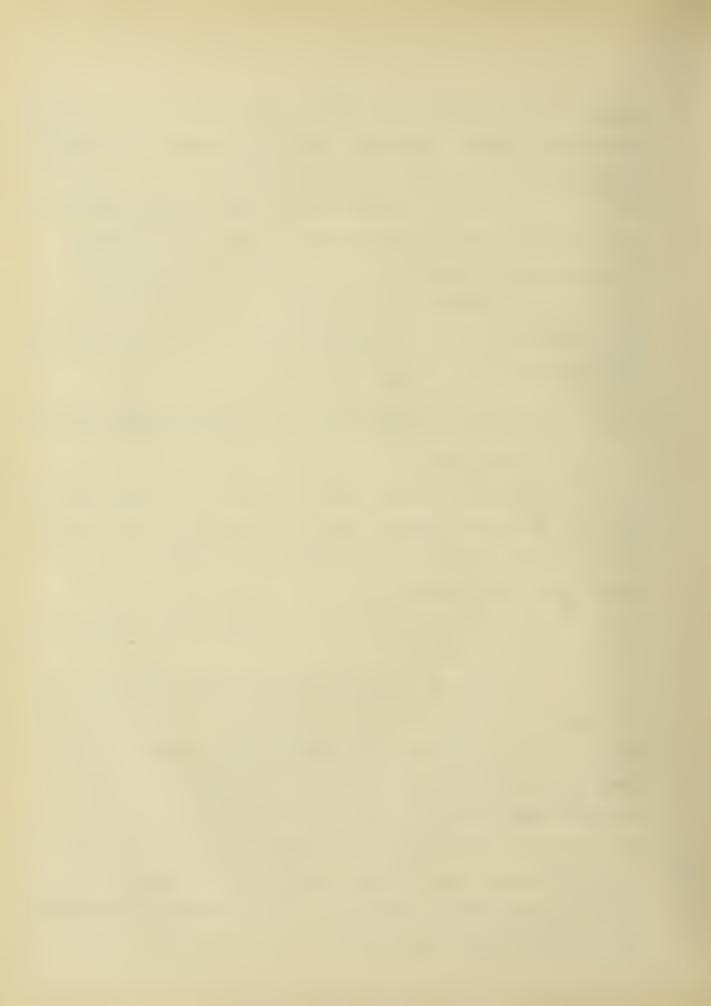
$$H = \frac{41 \text{ V} - 6000 \text{ x} 20.5}{11.8} = \frac{41 \text{ x} 4500 - 6000 \text{ x} 20.5}{11.8} = 5340 \text{ lb.}$$

At this point the line of pressure for dead load and therefore the center line of the arch is about 9.7 feet above a horizontal line through the skewback hinges. The maximum positive moment will therefore be

$$+M = 4500 \times 20.5 - 5340 \times 9.7 = 92400 - 51800 = 40600 lb.$$

ft. = 487200 lb.-in.

The maximum negative bending moment will occur when the concentrated load is at the crown, and will be at the point where the line of pressure and therefore the center line of the arch, deviates most from a straight line through the crown and skewback hinges, since this line represents the direction of the reaction due to the concentrated load alone. By scaling from the drawing this was found to be at a point 185 feet from the skewback where the line of pressure is 9.3 feet above the horizontal through the skewback hinge.



By the same method of computation as used before,

$$V = 3000 \text{ lb.}$$
 $H = 10680 \text{ lb.}$

$$M = 3000 \times 18.5 - 10680 \times 4.3 = -44000 \text{ lb. ft.} = -528000 \text{ lb. in.}$$

Therefore the maximum thickness of the arch is required at a point about 18.5 feet from the shrewback hinges. To determine the position of the neutral axis the following formula, given in Taylor and Thompson's Joncrete Plain and Reinforced, p. 564, was used.

$$x = \sqrt{2r(p+p'a)+r^2(p+p')^2} - r(p+p')$$

where x = ratio of depth of neutral axis to depth of steel in tension.

- r = ratio of modulus of elasticity of steel to modulus of elasticity of concrete.
- p' = ratio of cross section of steel in compression to to cross section of beam between steel in tension and outer fiber in compression.
- p = ratio of cross section of steel in tension to same cross section of concrete.
- a = ratio of depth of steel in compression.

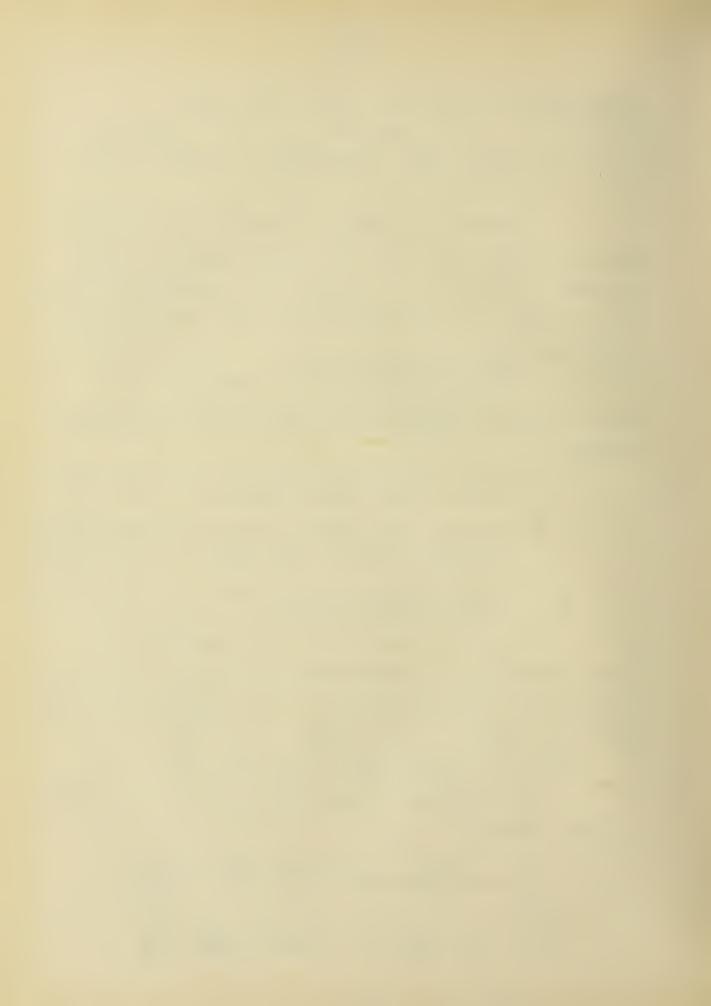
It was assumed that the reinforcement would be 1/4 of one percent on each side of the neutral axis and r was assumed as 20. This gave, p = .0025 p' = .0025 a = 1/12

x = .24 or say 1/4 which is the value used.

The following formulas are also given by the authority above mentioned.

$$M = Cbd^{2} \left[\frac{x}{2} (1 - \frac{x}{3}) + \frac{rp'(1 - a)(x - a)}{x} \right]$$

$$M = Sbd^{2} \left[p(1-a) - \left(\frac{x^{2}}{2r(1-x)} \right) \left(\frac{x}{3} - a \right) \right]$$



in which M = moment of resistance in lb. in.

C = compression in concrete in 1b. per sq. in.

S = stress in steel in tension lb. per sq. in.

b = width of section (12 inches for this case)

d = depth of section below steel in inches.

Substituting in the above formulas,

$$M = 12 \text{ Cd}^{2} \left[\frac{1}{8} (1 - \frac{1}{12}) \right] 20 \text{ x } .0025 \text{ x} \frac{1}{6} \text{ x } \frac{11}{12} \text{ x } 4$$

$$= 1.741 \text{ Cd}^{2}$$

$$C = \frac{M}{1.741 \text{ d}^{2}}$$

$$M = 12 \text{ Sd}^{2} \left[.0025 \text{ x} \frac{11}{12} - \frac{.0625}{40 \text{ x} \frac{11}{12}} \right] \left(\frac{1}{12} - \frac{1}{12} \right)$$

$$= .0275 \text{ Sd}^{2}$$

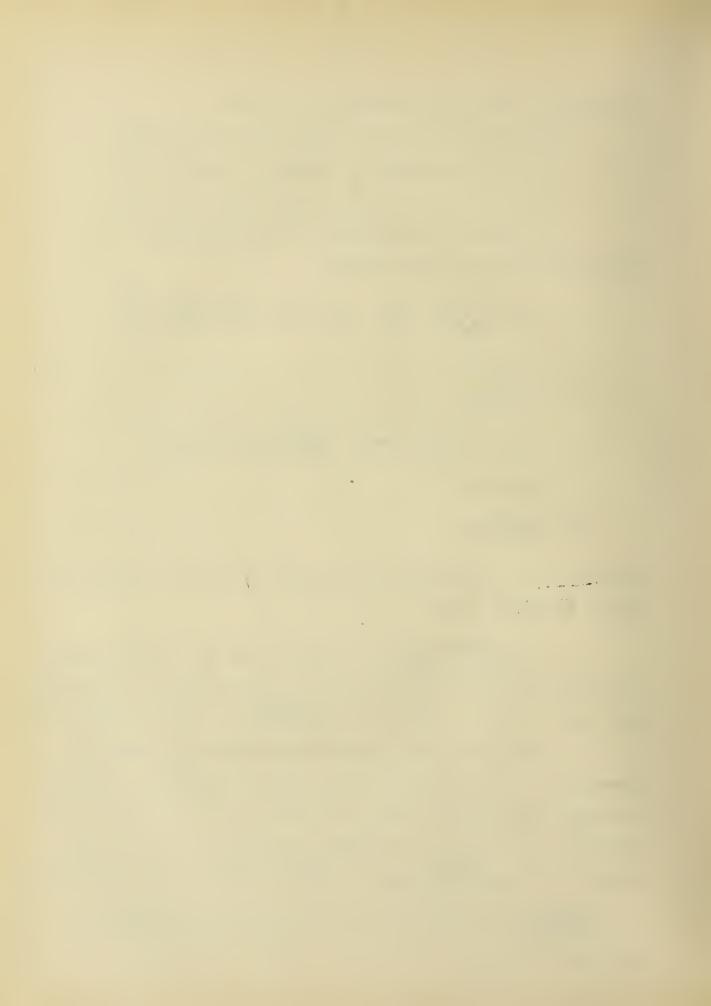
$$S = \frac{M}{.0275 \text{ d}^{2}}$$

Assuming d = 26 inches the thickness of the arch would be 28 inches for which values

 $C = \frac{528000}{1.741 \text{ x } (26)^2} = 448 \text{ lb. per sq. in. compression}$ in extreme fiber of concrete, due to bending moment. The direct dead load stress = 38700 lb. or $\frac{38700}{12 \text{ x } 28} = 115 \text{ lb. per sq. in.}$

The total compressive stress therefore would be 448 + 115 = 563 lb. per sq. in. which is excessive since the allowable compressive stress in concrete is 500 lb. per sq. in. Assuming d = 28 in.the thickness would be 30 in. for which values C = $\frac{528000}{1.741 \text{ x } (28)^2}$ = 387 lb. per sq. in. compression.

 $\frac{38700}{12 \times 30}$ = 108 lb. per sq. in. direct compression in concrete.



The total compressive stress would be 387 + 108 = 495 lb. per sq. in. This stress is due to the live load bending stress and the dead load direct stress. The live load direct stress was about 15 lb. per sq. in. but was neglected because the direct stress due to dead load is greater than it will be after the section is reduced and is probably more than it will be for the distresses rect dead and live load, when the section is revised.

$$M = .0275 \text{ Sd}^2$$
 $S = \frac{M}{.0275 \text{ d}^2}$

 $S = \frac{528000}{.0275 \times (28)^2} = 24400 \text{ lb. per sq. in. tension}$ in steel due to bending moment.

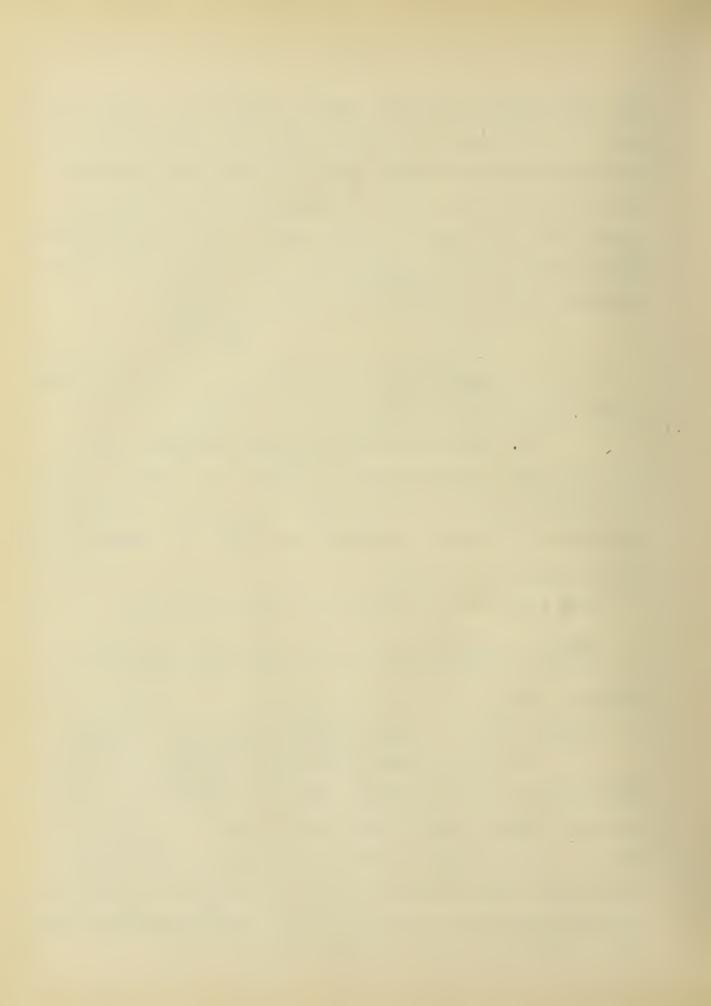
But under the assumption that the neutral axis is that above the middle of the section, and there is no tension in concrete, more than 1/2 the direct stress will tend to compress the steel that is in tension, or more than $\frac{38700}{2}$ = 19850 lb. tends to put compression in steel.

.0025 x 12 x 28 = .84 sq. in. of steel in tension

19850 = 23600 lb. per sq. in. compression in steel due to direct stress.

24400 - 23600 = 800 lb. per sq. in. tension in steel.

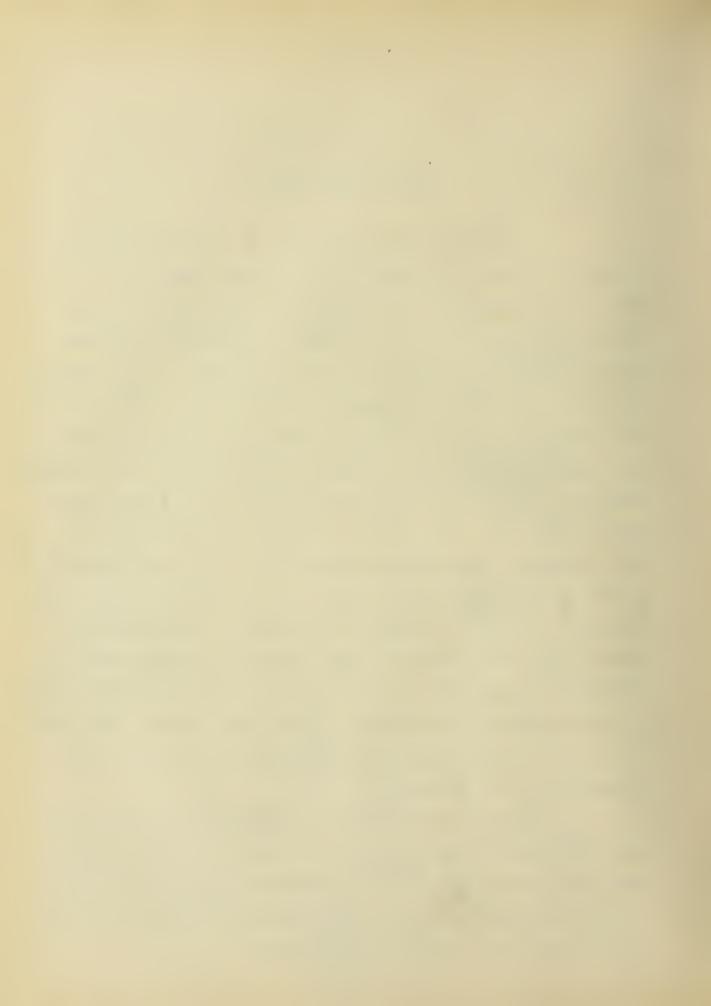
The steel seemed, therefore, to be excessive but if the percent of steel be farther reduced it will make the neutral axis move further from the steel and will require a heavier section. It did not seem advisable to allow the neutral axis to move further from the tension side so a section of 30 inches in depth was chosen for the point 18.5 feet from the skewback hinge for the next trial arch to be investigated.



SECOND TRIAL ARCH.

An arch was next drawn having a thickness of 14 inches at the crown, 30 inches at the quarter points and 16 inches at the skewbacks. These dimensions were laid out on the drawing and a three-centered intrados and extrados were drawn in such a manner as to keep the center line close to the line of pressure. The earth was assumed one foot above the crown on this arch instead of 6 inches as in the first arch. This arch was then drawn on another plate (Plate 2) and together with the superincumbant earth it was divided into sections in the same manner as the first arch. The center of gravity of each section was then determined graphically considering the earth and concrete to form one homogenous mass. This is not strictly true but will not affect the line of action of the forces to any appreciable amount. Then the distances of these centers of gravity from the skewbacks were measured. These are the moment arms of the forces W and are marked X in the table. WX was next computed and found as for the previous arch. The new line of pressure was then placed on the drawing.

The dead load moment at section 18.3 feet from the skewback hinge was next computed. In order to do this the disthrough tance from a horizontal , the skewback hinges to the center of the section was measured. This was found to be 9.3 feet and is the moment arm of H about that section.



 $\rm M_{
m d}$ = 18.3 V - 9.3 H - $\rm \xi wx$ where x is the moment arm of each of the weights to the left of the section and w is the corresponding weight.

$$M_d$$
 = 18.3 x 28300 - 9.3 x 34600 - (3930 x 16.85 + 3795 x 13.8 + 5350 x 9.95 + 4325 x 5.0)

$$= +1600 \text{ lb. ft.}$$
 = 19200 lb.-in.

To determine the live load moment, let the horizontal distance from the skew to any section be (a) and the vertical distance from a horizontal through the skewback hinges to the center of the section be called (b). Then for a concentrated live load of 6000 lb. at any point <u>a</u> feet from the skewback hinge the maximum positive moment will occur under the load and

$$V_{c.1.} = \frac{6000(82 - a)}{82} = 6000 - 73.15 a.$$
 (1)

$$H_{c.1.} = \frac{73.15 \times 42 \text{ a}}{11.8} = 254.5 \text{ a}.$$
 (2)

$$+M_{c.1.} = Va - Hb = 6000 a - 73.15 a^2 - 2545 ab.$$
 (3)

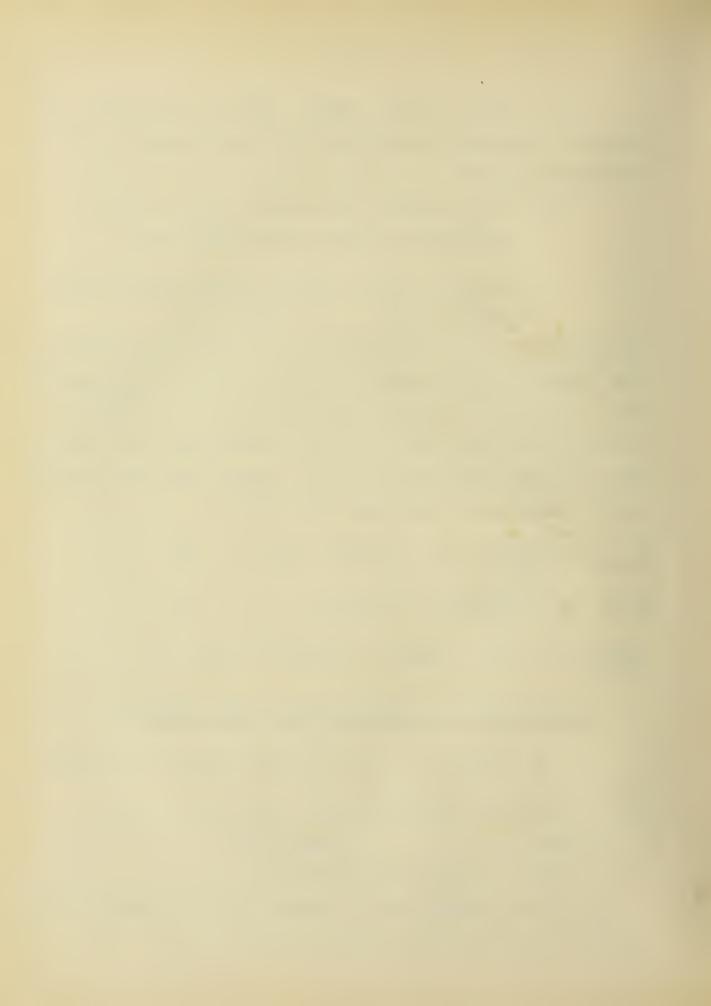
Investigation of section 18.3 feet from Skewback.

$$M_1 = 6000 \times 18.3 + 73.15 \times (18.3)^2 - 254.5 \times 9.3 \times 18.3$$

= +42000 lb. ft. = +504000 lb. in.

$$M = +504000 + 19000 = +523000$$
lb. in.

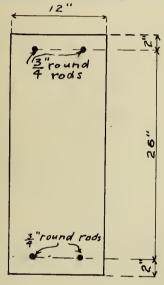
The maximum negative moment will occur when the concentrated load of 6000 lb. is at the crown.



V = 3000 lb. H = 10680 lb.

 M_1 = Va - Hb = 3000 x 18.3 - 10680 x 9.3 = -44300 lb. ft. = -531000 lb. in.

- M = 531000 + 19000 = -51200 lb. in. which is the maximum negative bending moment at the section considered. Therefore the maximum bending moment in the section is +523000 lb. in.



The section is 30 inches deep and reinforced as shown in figure.

28 x 12 = 336 sq. in. of concrete above steel.

.0025 x 336 = .84 sq. in. of steel required for each foot of width of arch. Using two 3/4-inch round rods leaving an area of 0.44 sq. inches in each side of section.

 $\frac{0.88}{336}$ = .00262 = proportion of steel = p = p'.

According to Taylor and Thompson,

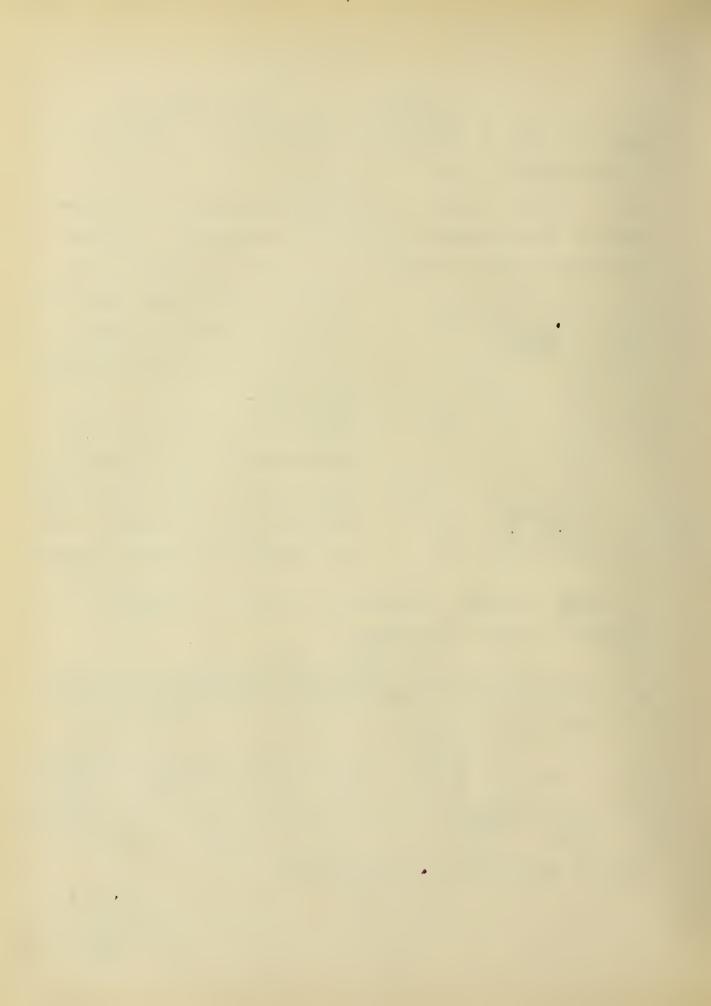
$$a = \frac{2}{28} = .068.$$

$$x = \sqrt{2 \times 20(.00262 + .00267 \times .069) + (20)^2 (.00264 + .0062)^2} - 20(.00262 + .00262)$$

$$M = C 12 \times (28)^{2} \left[\frac{.246}{2} (1 - \frac{.246}{3}) + \frac{20 \times .00262(.246 - .069)(1 - .069)}{.246} \right]$$

 $C = \frac{M}{1390} = \frac{523000}{1390} = 375 \text{ lb. per sq. in. compression in extreme}$

fiber of concrete due to bending moment.



The Direct Stress in the Section.

The vertical component of the direct stress, or the vertical shear v, in the section is equal to the vertical component of the left section V minus the loads to the left of the section.

$$V_d$$
 = 28300
 V_1 = 6000 - 73.15 a (See equation 1)
 V = 28300 + 4660 = 32960 lb.

The loads to left of section are,

$$3930 + 3795 + 3350 + 4325 + 1/2 \times 3425 = 19112$$

v = $32960 - 19112 = 13848$ or 13850 lb.

The horizontal component, h, of the direct stress in the section is equal to the dead load horizontal thrust (H_d) plus the live load thrust (H_1) .

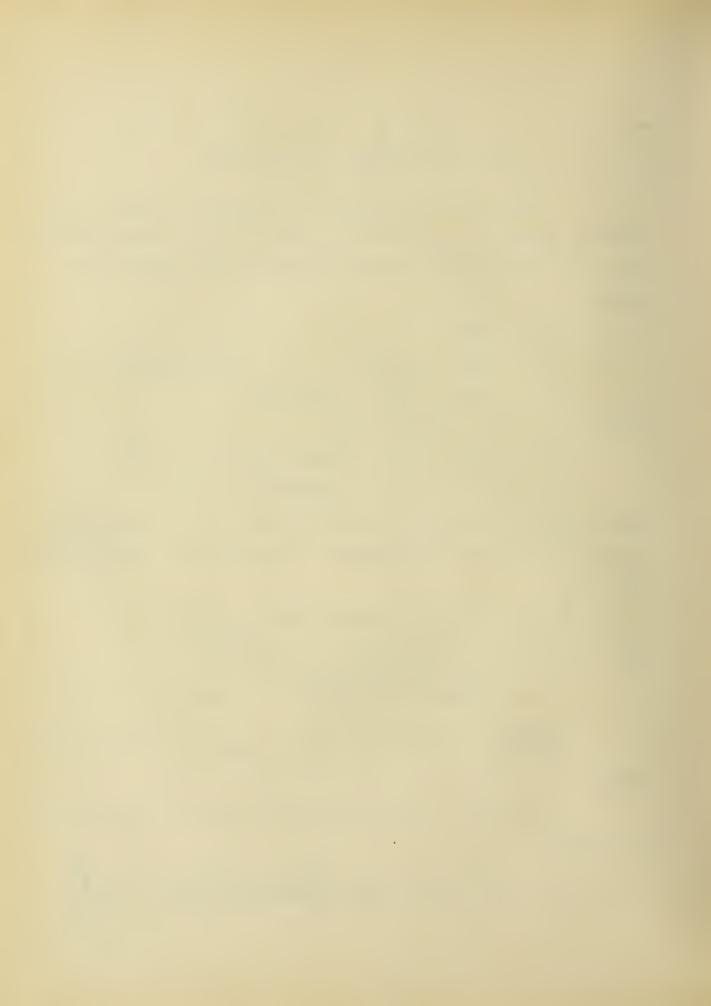
$$h = H_d + H_1 = .34600 + 4660 = .39260.$$
 The resultant direct compression D = $\sqrt{v^2 + h^2}$

$$D = \sqrt{(13850)^2 + (39260)^2} = 41600 \text{ lb.}$$

 $\frac{41600}{12 \times 30}$ = 715 lb. per sq. in. compression due to direct stress.

375 + 115 = 490 lb. per sq. in. maximum compression in concrete.

$$M = S \times 12 \times (28)^{2} \left[.00262(1-.069) - \left(\frac{(.246)^{2}}{40(1-.246)} \right) \left(\frac{.246}{3} - .069 \right) \right]$$



$$\frac{M}{S} = 9400 \left[.00262 \times .931 - \frac{(.246)^2}{40 \times .754} \times .013 \right]$$

 $= 9400(.00244 - .000025) = 9400 \times .00241 = 22.65$

S = $\frac{M}{22.65} = \frac{523000}{22.65} = 23000$ lb. per sq. in. tension in steel due to bending moment.

But under the assumption that the neutral axis is above the middle of the section, more than 1/2 the direct stress will tend to compress the steel that is in tension or more than $\frac{41600}{2} = 20800$ lb. tends to compress this steel. But $\frac{20800}{.88} = 23600$ lb. per sq. in. compression in steel which shows that the neutral axis will not rise so high. But if the neutral axis is nearer the tension side the tension in the steel will be greater than indicated. Suppose that x should become as large as 0.5 and for simplicity assume the center of the compressive force in the steel in compression to be at the center of compressive forces in the concrete, which is on the safe side. Then taking moments about this center of compressive forces, the moment arm of the steel in tension equals 5/6 d and M = 5/6 bd 2S or $S = 1.2 \frac{M}{PbdZ}$.

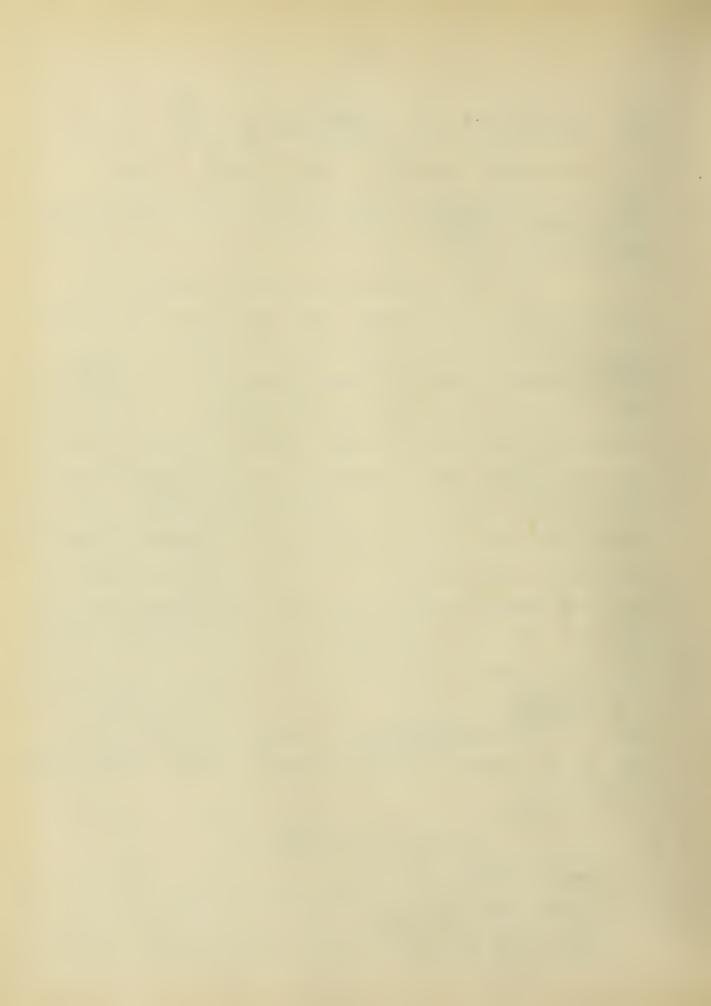
Here S = $1.2 \times \frac{523000}{9400 \times .0026}$ = 26700 lb. per sq. in. In that case the tension in the steel would be 26700 - 23600 = 3100 lb. per sq. in.

Investigation for Shear.

Vertical depth = 32.4 in.

Vertical area = 388 sq. in.

 $\frac{13850}{388}$ = 36 lb. per sq. in. shear in concrete.



Investigation of Section 1.45 Feet from Skewback.

a = 1.45 b = 1.1

 M_d = 28300 x 1.45 - 34600 x 1.1 - 1960 x 0.65 = 1600 lb.ft. + M_1 = 6000 x 1.45 - 73.15 x(1.45)² - 254.5 x 1.45 x 1.1 = 8140 lb. ft.

 $-M_{1}$ = 3000 x 1.45 - 10680 x 1.1 = -7400 lb. ft.

Maximum moment = 8140 + 1600 = +9740 lb. ft. = +117000 lb.in.

Assuming section 19 inches deep,

 $C = \frac{M}{628} = \frac{117000}{628} = 186 \text{ lb. per sq. in.}$

 $h = 34600 + 254.5 \times 1.45 = 34969$

 $v = 28300 + 6000 - 73.15 \times 1.45 - 1/2 \times 3930 = 32230 \text{ lb.}$

 $D = \sqrt{(35000)^2 + (32000)^2} = 47500 \text{ lb.}$

 $\frac{475000}{12 \times 19} = 208 \text{ lb. per sq. in. due to direct stress.}$

208 + 186 = 394 lb. per sq. in. total.

Investigation of Section 4.5 Feet from Skewback.

a = 4.5 b = 3.25

 M_{d} = 28300 x 4.5 - 34600 x 3.25 - 3930 x 3.05 - 1/2 x 3795 x .75 = + 1100 lb. ft.

 $+ M_1 = 6000 \times 4.5 - 73.15 \times (45)^2 - 254.5 \times 4.5 \times 3.25$ = 27000 - 1495 - 3720 = 21800 lb. ft.



 $-M_1$ = 3000 x 4.5 - 10680 x 3.25 = 13500 - 34700 = -21200 Maximum moment = 21800 + 1100 = 22900 lb. ft. = 275000 lb.in. Section is 25 inches deep but assume 24 inches, then,

 $C = \frac{275000}{921} = 295 \text{ lb. per sq. in. due to bending moment.}$

 $h = 34600 + 254.5 \times 4.5 = 35745 lb.$

 $v = 28300 - 6000 - 73.15 \times 4.5 - 3930 - 1/2 \times 3795 = 28144$

 $D = \sqrt{(28100)^2 + (35700)^2} = 45500 \text{ lb.}$

 $\frac{45500}{12 \times 24}$ = 158 lb. per sq. in.

215 + 158 = 453 lb. per sq. in. total compression.

Investigation of Section 8.45 Feet from Skewback.

a = 8.35 ft. b = 5.5 ft.

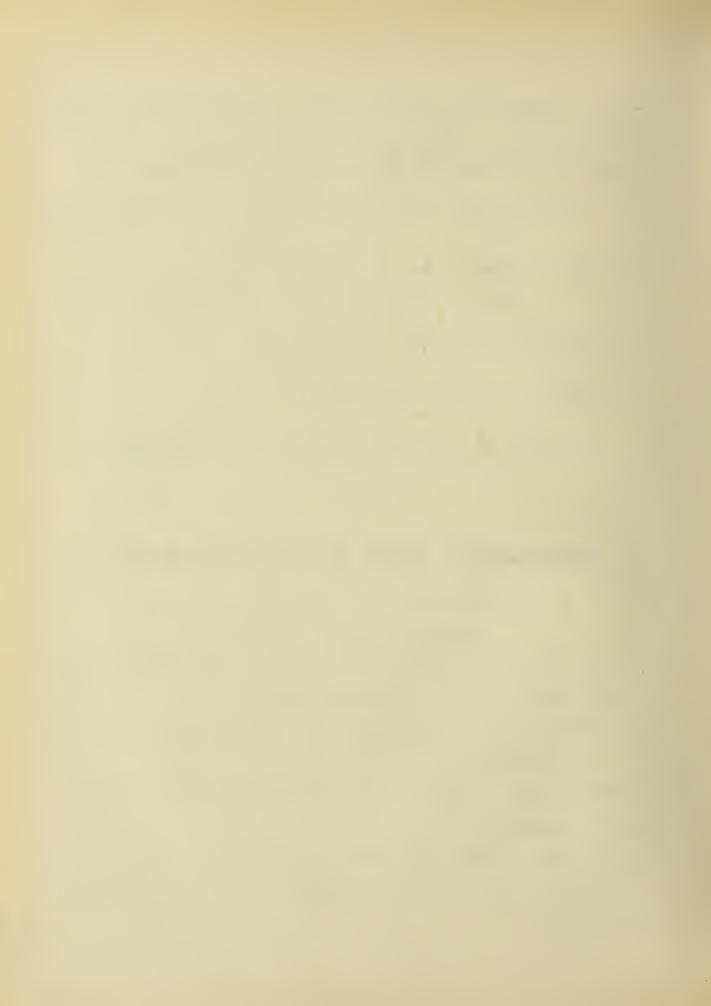
depth of section = 30 inches.

 M_d = 8.35 x 28300 - 5.5 x 34600 - (3930 x 6.9 + 3795 x 3.85) =+ 4100 lb. ft. = + 49200 lb. in.

 $+ M_1 = 6000 \times 8.35 - 73.15 \times (8.35)^2 - 254.5 \times 8.35 \times 5.5$ = + 33350 lb. ft.

33350 + 4100 = 37450 lb. ft. = 449500 lb. in. maximum positive bending moment.

 $-M_1$ = 3000 - 8.35 - 10680 x 5.5 = -33650 lb. ft. -33650 + 4100 = -29450 lb. $^{ft}_{\Lambda}$ = 353500 lb. in. maximum negative bending moment.



Therefore 449000 lb. in. is maximum moment.

 $C = \frac{M}{1672} = \frac{449500}{1672} = 269 \text{ lb. per sq. in. due to}$ bending moment.

 $V_d = 28300 \text{ lb.}$

 V_1 = 6000 - 73.15 x 8.35 = 6000 - 612 = 5388 lb. V = 28300 + 5388 = 33688 lb.

 $\mathbf{E}_{W} = 3930 + 3795 + 1/2(5350) = 10400 \text{ lb.}$

 $v = V - \Sigma_W = 23288 \text{ lb.}$

 $H_{d} = 34600 \text{ lb.}$

 $H_{1} = 454.5 \times 8.35 = 3800 \text{ lb.}$

h = 34600 + 3800 = 38400 lb.

 $D = \sqrt{(38400)^2 + (23288)^2} = 48400 \text{ lb.}$

 $\frac{44800}{12 \times 30}$ = 125 lb. per sq. in. compression due to direct stress.

269 + 125 = 394 lb. per sq. in. total compression. Since the tensile stress in the steel will be less than in the previous section it need not be investigated.

Investigation for Shear.

Vertical depth of section = 33.6 in.

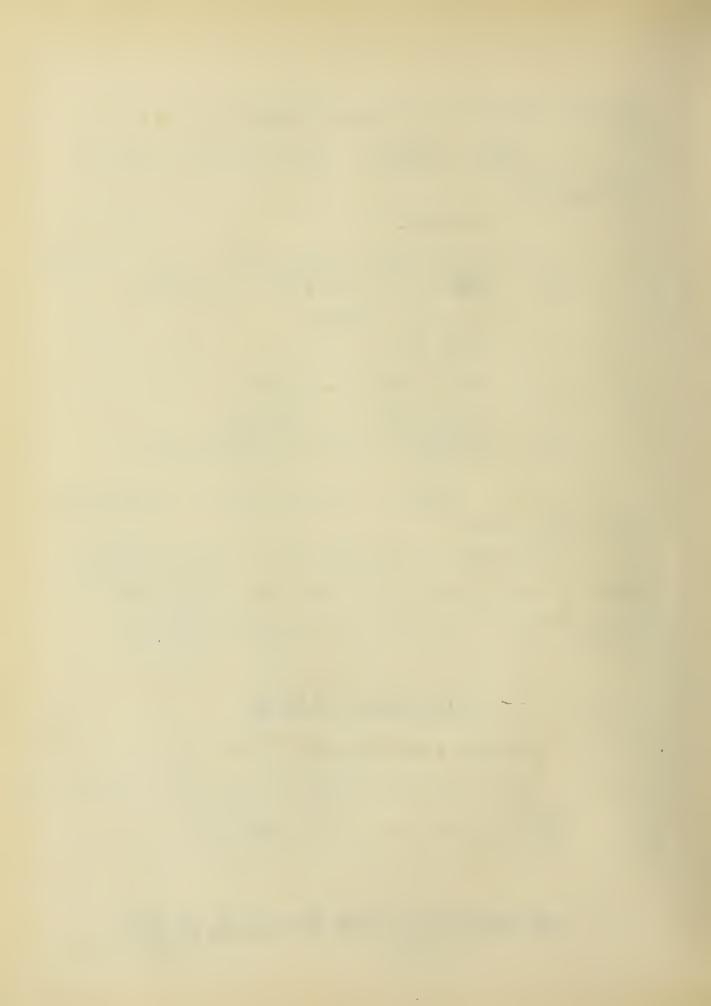
Area of vertical section = $33.6 \times 12 = 403 \text{ sq. in.}$

= 58 lb. per sq. in. shear in concrete.

Investigation of Section 31 Feet from Skewback.

a = 31 feet

b = 11.4



$$M_{d}$$
 = 31 x 28300 - 11.4 x 34600 - (3930 x 29.55 + 3795 x 26.5 5350 x 22.65 + 4325 x 17.7 + 3425 x 12.7 + 4650 x 5.5)

= 878000 - 394000 - 116000 - 100800 - 121000 - 76600 - 43500 - 25600 = + 500 lb. ft. = + 6000 lb. in.

 $+ M_1 = 6000 \times 31 - 73.15 \times (31)^2 - 254.5 \times 31 \times 11.4$

= 186000 - 70500 - 89900 = +25600 lb. ft. =+307200 lb.in.

 $-M_1 = 3000 \times 31 - 10680 \times 11.4 = -28800$ lb. ft.

Maximum moment at this section is therefore,

 $M_d + M_1 = 6000 + 307200 = +313200 lb. in.$

Depth of section = 19 inches

Area of concrete above steel in tension = 17 x 12 = 204 sq.in.

$$\frac{.88}{204} = .00431 = p = p'$$

$$a = \frac{2}{17} = .1175$$

$$\mathbf{x} = \sqrt{2 \times 20(.00431 + .00431 \times .1175) + (20)^{2}(.00431 + .00431)^{2}}$$

$$- 20(.00431 + .00431)$$

= .300

$$M = C \times 12 \times (17)^{2} \left[\frac{.30}{2} (1 - \frac{.30}{3}) + \frac{20 \times .00431(.3 - .118)(1 - .118)}{.30} \right]$$

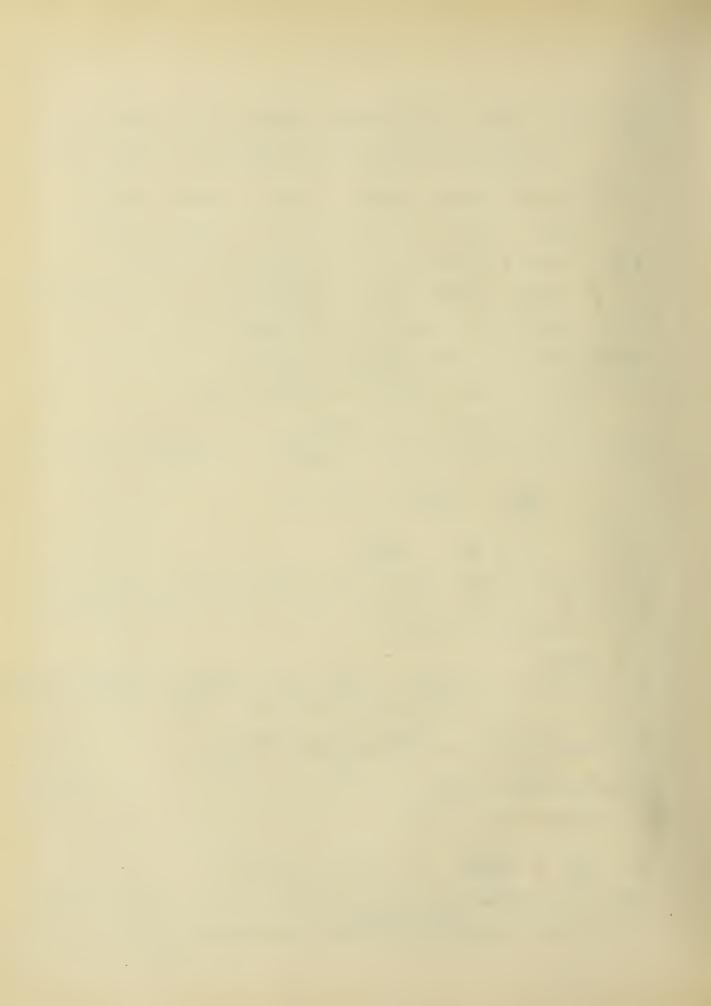
$$\frac{M}{C} = 3470 \left[.15 \times .9 + \frac{.0862 \times .182 \times .882}{.30} \right]$$

= 3470(.135 + .0461)

 $= 3470 \times .1811 = 628$

$$C = \frac{M}{628} = \frac{313200}{628} = 498 \text{ lb. per sq. in.}$$

Thus the allowable compressive stress is reached without considthe section ering direct stress and is therefore too shallow.



Next a section 22 inches deep was tried.

Area of concrete above lower steel = 12 x 20 = 240

sq. in.

$$\frac{.88}{240} = .00366 = p = p'$$

$$a = \frac{2}{20} = 0.1$$

$$x = \sqrt{2 \times 20(.00366 + .1 \times .00366) + (20)^{2}(.00366 + .00366)^{2}}$$

$$- 20(.00366 + .00366)$$

= .281

$$M = C \times 12(20)^{2} \left[\frac{.281}{2} (1 - \frac{.281}{3}) + \frac{20 \times .00366(.281 - .1)(1 - .1)}{.281} \right]$$

= 814

$$C = \frac{M}{814} = \frac{313200}{814} = 385$$
 lb. per sq. in. due to bending moment.

 $H_d = 34600 \text{ lb.}$

 $H_7 = 254.5 \times 31 = 7880 \text{ lb.}$

h = 34600 + 7880 = 42480 lb.

 $V_{d} = 28300 \text{ lb.}$

 $V_{3} = 6000 - 73.15 \times 31 = 6000 - 2270 = 3770 lb.$

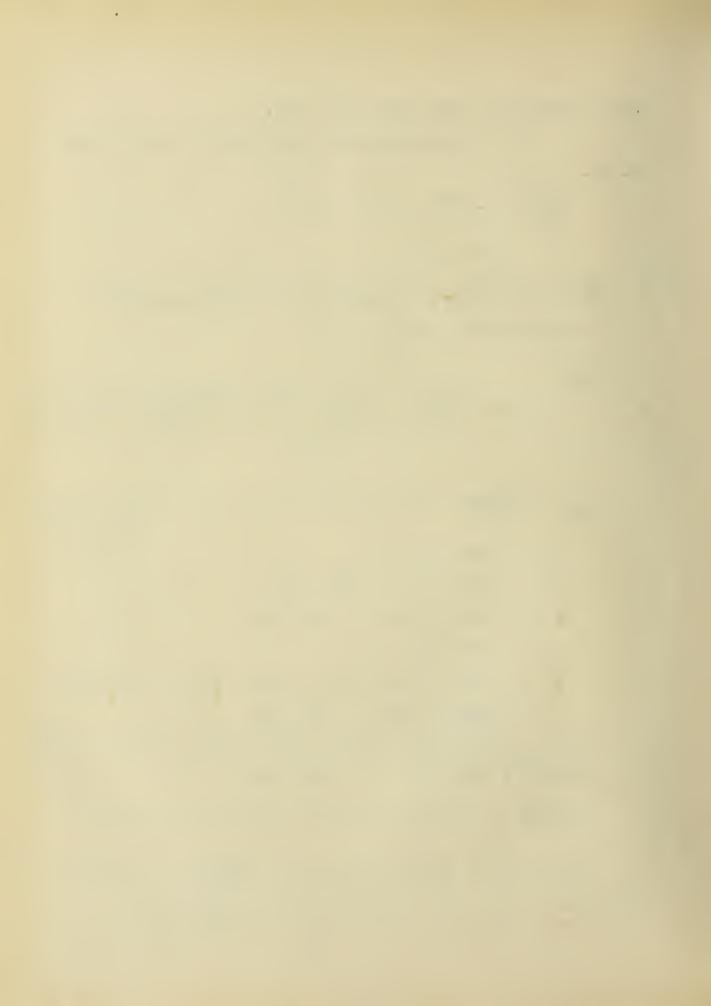
V = 28300 + 3770 = 32070 lb.

v = 32070 - (3930 + 3795 + 5350 + 4325 + 4325 + 4650) = 7000

 $D = \sqrt{(7000)^2 + (42500)^2} = 43200 \text{ lb.}$

 $\frac{43200}{12 \times 22}$ = 164 lb. per sq. in. due to direct stress.

385 + 164 = 549 lb. per sq. in. compression, which is excessive and shows that a deeper section is required.



Then a section 24 inches deep was tried.

Area of concrete above lower steel = $12 \times 22 = 264$ sq. in.

$$\frac{.88}{264} = .00333 = p = p'$$

$$a = \frac{2}{22} = .091$$

$$\mathbf{x} = \sqrt{2 \times 20(.00333 + .091 \times .00333 + (20)^{2} (.00333 + .00333)^{2}}$$

$$- 20(.00333 + .00333)$$

= .2633

$$\frac{M}{C} = 12 \times (22)^2 \left[\frac{.26}{2} (1 - .083) + \frac{20 \times .00333(.26 - .091)(1 - .091)}{.26} \right]$$

= 921

 $C = \frac{313200}{921} = 340 \text{ lb. per sq. in. due to bending moment.}$ $\frac{43200}{12 \times 24} = 150 \text{ lb. per sq. in. due to direct stress.}$

340 + 150 = 490 lb. per sq. in. total compressive stress in concrete.

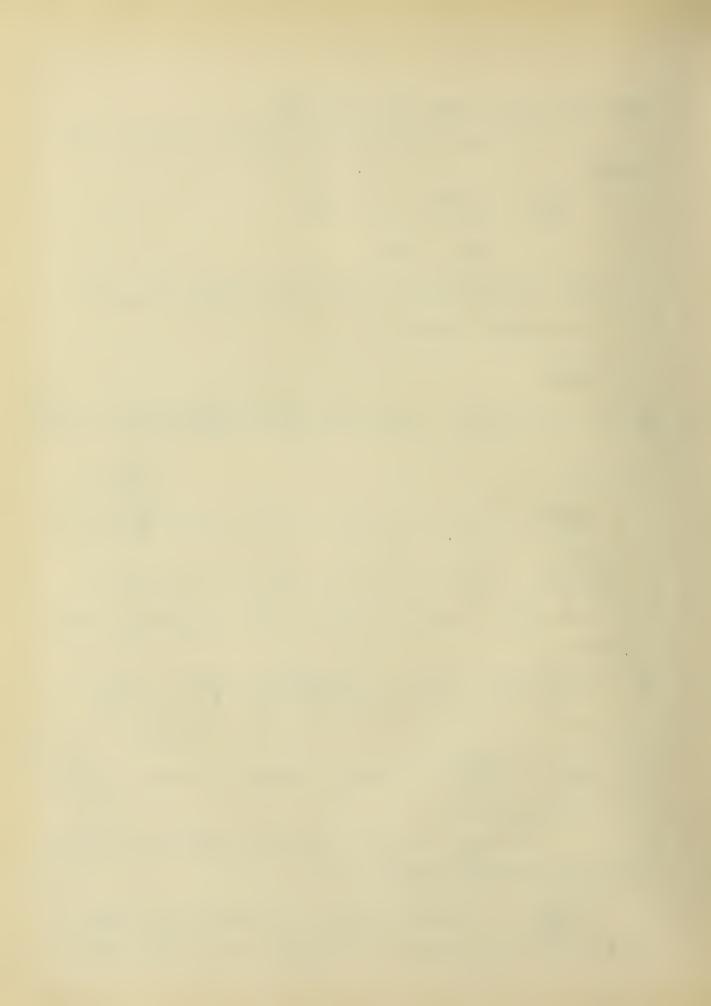
$$\frac{M}{S} = 5800 \left[.00333(1 - .091) - \frac{(.26)^2}{2 \times 20(1 - .26)} \left(\frac{.26}{3} - .091 \right) \right]$$

$$= 17.7$$

S = $\frac{11}{17.7}$ = $\frac{313200}{17.7}$ = 17700 lb. per sq. in. tension in steel due to bending moment.

But there is at least 22350 lb. compression in steel in tension side due to direct stress.

 $\frac{22350}{.88}$ = 25700 lb. per sq. in. compression in steel in tension side with neutral axis as assumed, which shows that neu-



tral axis will not be as far from tension side as assumed.

 $\frac{7000}{12 \times 24}$ = 24 lb. per sq. in. shear in concrete.

Therefore a section 24 inches deep was used for next trial.

Investigation of Section 35.7 Feet from Skewback.

Taking moment of forces to right of section, $M_d = H_d \times (11.8 - b) - 1/2 \times 2850 \times \frac{5.3}{2}$

 $M_d = 34600 \times 0.05 - 3780 = 1730 - 3780 = -2050 lb. ft.$

 $+M_1$ = 6000 x 35.7 - 73.15x(35.7)² - 254.5 - 35.7 - 11.75 = +15300 lb. ft.

 $- M_{1} = 3000 \times 35.7 - 10680 \times 11.75 = -18300 lb. ft.$

-18300 - 2050 = -20350 lb. ft. = -246500 lb. in.

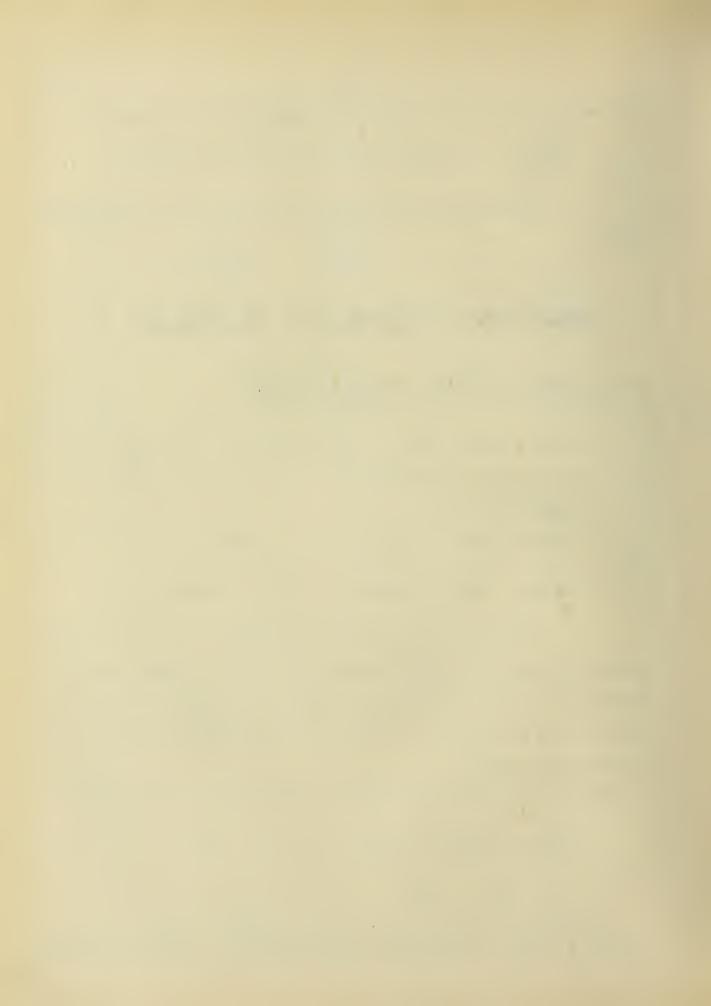
Section is 16 inches deep on arch on Plate 2 but on account of having to make section at a=31, 24 inches deep, a section at a=36 of 18 inches depth was assumed, as the deepening of the section at a=31 will deepen the section at a=36 about this much.

Area of concrete above steel = 16 x 12 = 192 sq. in.

$$\frac{.88}{192} = .00458 = p = p'$$

$$a = \frac{2}{16} = .125$$

 $x = \sqrt{40 \times 1.125 \times .00458 + 400 \times (2 \times .00458)^2} - 20 \times 2 \times .00458$ = .31



$$\frac{M}{C} = 12 \times (16)^2 \left[\frac{.31}{2} (1 - .103) + \frac{20 \times .00458(.31 - .125)(1 - .125)}{.31} \right]$$

= 573

 $C = \frac{M}{573} = \frac{246500}{573} = 430 \text{ lb. per sq. in. due to bending.}$

 $V_d = 28300 \text{ lb.}$

 $V_1 = 6000 - 73.15 \times 35.7 = 600 - 2610 = 3390 lb.$

 $V = 28300 \quad 3390 = 31690 \text{ lb.}$

 $v = 31690 - (3930 + 3795 + 5350 + 4325 + 3425 + 4650 + 1/2 \times 2850)$

= 4800 lb.

 $h = 34600 + 254.5 \times 35.7 = 43690 \text{ lb.}$

 $D = \sqrt{(43700)^2 + (4800)^2} = 44000 \text{ lb.}$

 $\frac{44000}{12 \times 18}$ = 204 lb. per sq. in. direct compression.

430 + 204 = 634 lb. per sq. in. total compression, which is excessive.

Next a depth of 19 inches was tried.

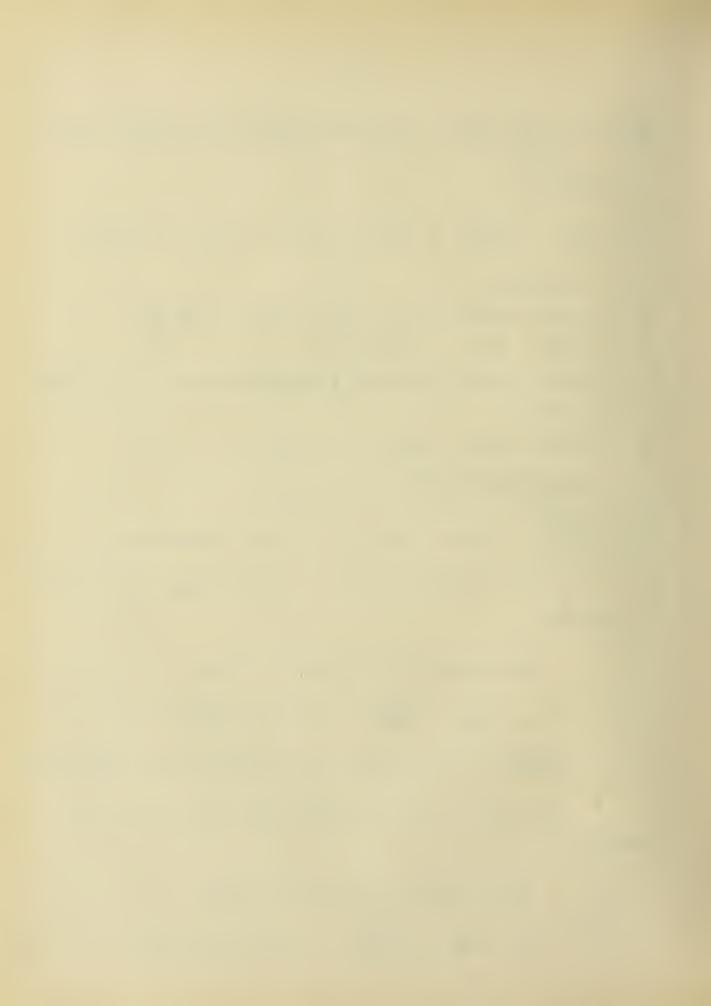
 $C = \frac{M}{628} = \frac{246500}{628} = 392 \text{ lb. per sq. in.}$

 $\frac{44000}{12 \times 19}$ = 193 lb. per sq. in. due to direct compression

342 + 193 = 585 lb. per sq. in. total, which is excessive.

Next a depth of 22 inches was tried.

 $c = \frac{M}{814} = \frac{246500}{814} = 302 lb. per sq. in.$



 $\frac{44000}{12 \times 22}$ = 167 lb. per sq. in.

302 + 167 = 469 lb. per sq. in. total compressive stress in concrete.

So it was decided to use a section 22 inches deep.

Investigation of Section 38.5 Feet from Skewback.

a = 38.5 b = 11.8

 $M_d = -1/4 \times 2850 \times 1.25 = -890.0 lb. ft.$

+ M_1 = 6000 x 38.5 - 73.15 x(38.5)² - 254.5 x 38.5 x 11.8 = +6500 lb. ft.

 $-M_1 = 3000 \times 38.5 - 10680 \times 11.8$ = -10500 lb. ft.

M = -10500 - 890 = -11390 lb. ft. = -125000 lb. in.

A section 19 inches deep was next tried.

 $C = \frac{M}{628} = \frac{125000}{628} = 199 \text{ lb. per sq. in.}$

h = 34600 + 10680 = 45280 lb.

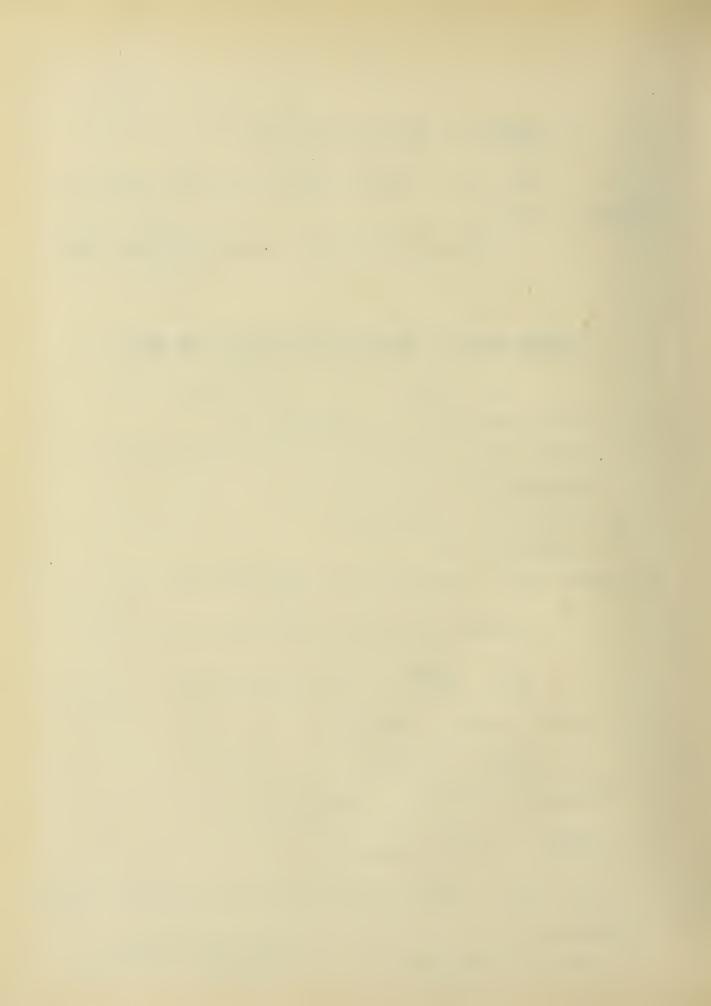
 $v = 1/4 \times 2850 = 712 \text{ lb.}$

 $D = \sqrt{(45300)^2 + (712)^2} = 45300 \text{ lb.}$

 $\frac{45300}{12 \times 19}$ = 198 lb. per sq. in.

199 + 198 = 397 lb. per sq. in. total compressive stress in concrete.

So a section 19 inches deep was used here and up to the crown.



H = 34600 + 10680 = 45280

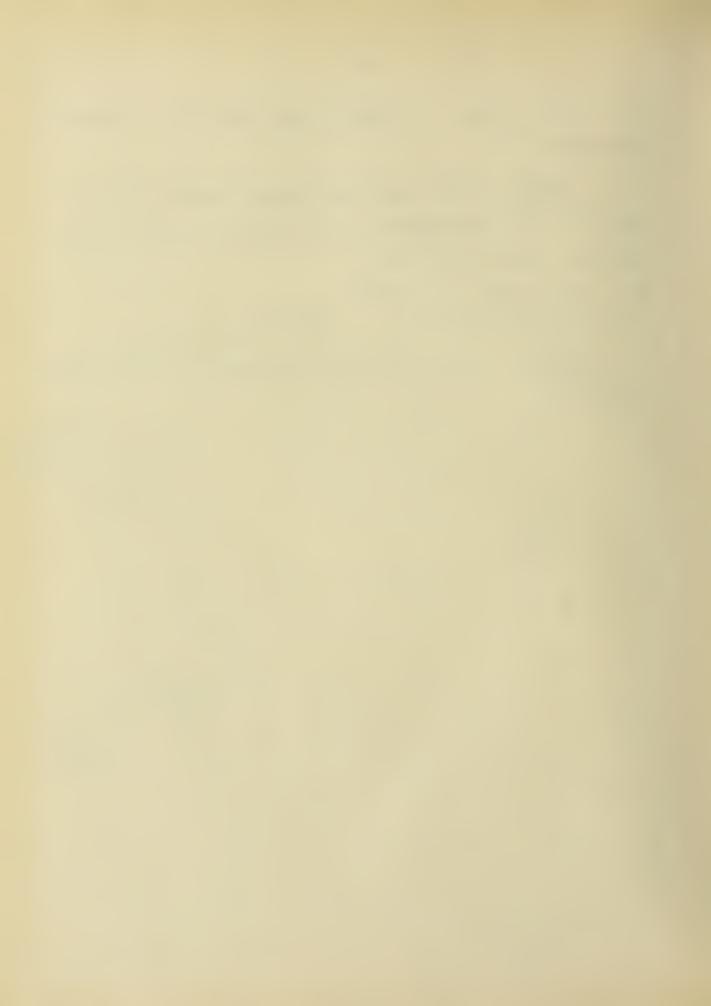
Let L = 34600 + 10680

Let L = 34600

 $\frac{45280}{12 \times 250}$ = 15 inches depth required under plate at crown for direct stress. But 19 inches was used on account of thickness required near crown.

V = 28300 + 3000 = 31300 lb. $R = \sqrt{(45300)^2 + (31300)^2} = 55000 lb.$

 $\frac{55000}{12 \times 250}$ = 18.3 inches depth required at skewback under hinge plate.



FINAL DESIGN.

An arch was next drawn having a thickness of 20 inches at the crown, 30 inches at the quarter points and 23 inches at the skewbacks. The intrados consists of an arc of a circle of 78 feet radius drawn to points 25 feet on each side of the crown and from there to the skewbacks arcs of 62.8 feet radius. The extrados consists of an arc of 129.0 feet radius drawn to points 15 feet on each side of the crown and from there to the skewbacks arcs of 50.7 feet radius. (See Plate 3).

The weights of earth and concrete were determined as before and different sections investigated.

Investigation of Section 4.5 Feet from Skewback.

a = 4.5 b = 3.05

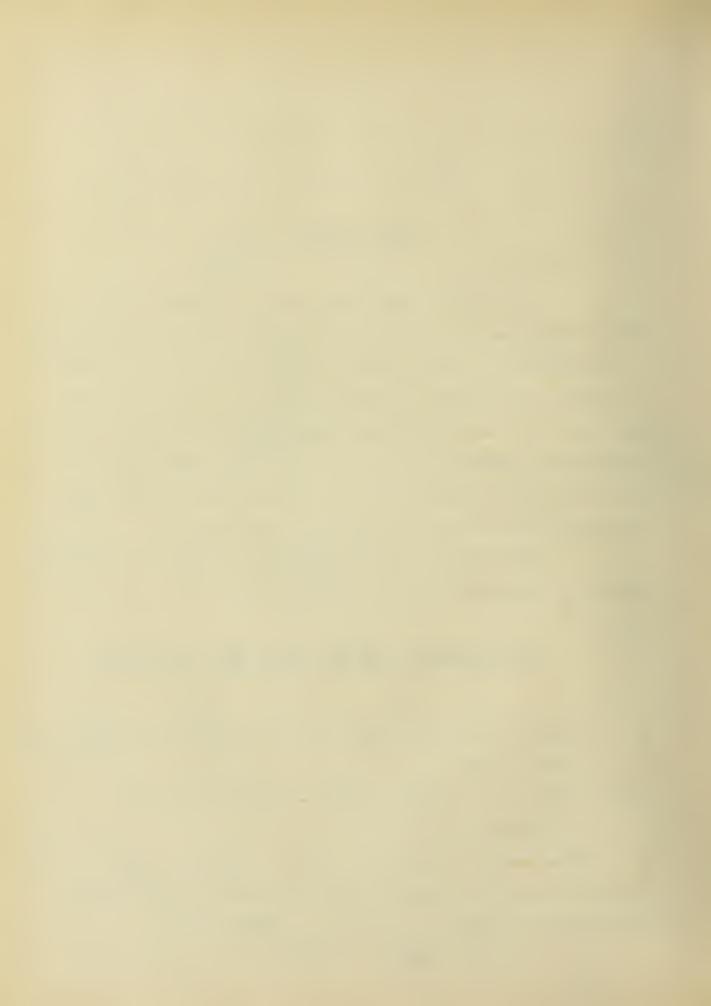
 M_{d} = 31280 x 4.5 - 39800 x 3.05 - 4510 x 3.05 - 1/2 x 3930 x .7 = +4000 lb. ft.

 $+ M_1 = 6000 \times 4.5 - 73.15 \times (4.5)^2 - 254.5 \times 4.5 \times 3.05$ = + 22000 lb. ft.

M = 22000 + 4000 = 26000 lb. ft. = 312000 lb. in.

The section here is 26 inches deep so the area of the concrete above steel in tension = $24 \times 12 = 288 \text{ sq. in.}$

$$p = p' = \frac{.88}{288} = .003055$$



$$a = \frac{2}{24} = .0834$$

$$x = \sqrt{40(.0031 + .0031 \times .0834) + 400 \times (2 \times .0031)^{2}} - 20 \times 2 \times .0031$$
= .261

$$\frac{M}{C} = 12 \times (24)^{2} \left[\frac{.261}{2} (1 - \frac{.261}{3}) + \frac{20 \times .0031 \times (.261 - .083)(1 - .083)}{.261} \right]$$

= 1091

 $C = \frac{M}{1091} = \frac{312000}{1091} = 286$ lb. per sq. in. compression in extreme fiber of concrete due to bending moment.

$$h = H_d + H_1 = 39800 + 254.5 \times 4.5 = 40900$$

$$v = 31280 + 6000 - 73.15 \times 4.5 - 4510 - 1/2 \times 3930 = 30480 lb.$$

$$D = \sqrt{(40900)^2 + (30500)^2} = 51200 \text{ lb. direct compression.}$$

 $\frac{51200}{12 \times 26}$ = 164 lb. per sq. in. compression due to direct stress.

286 + 164 = 450 lb. per sq. in. Λ compression in concrete at this section.

Investigation of Section 20.5 Feet from Skewback.

$$a = 20.5$$
 $b = 9.7$

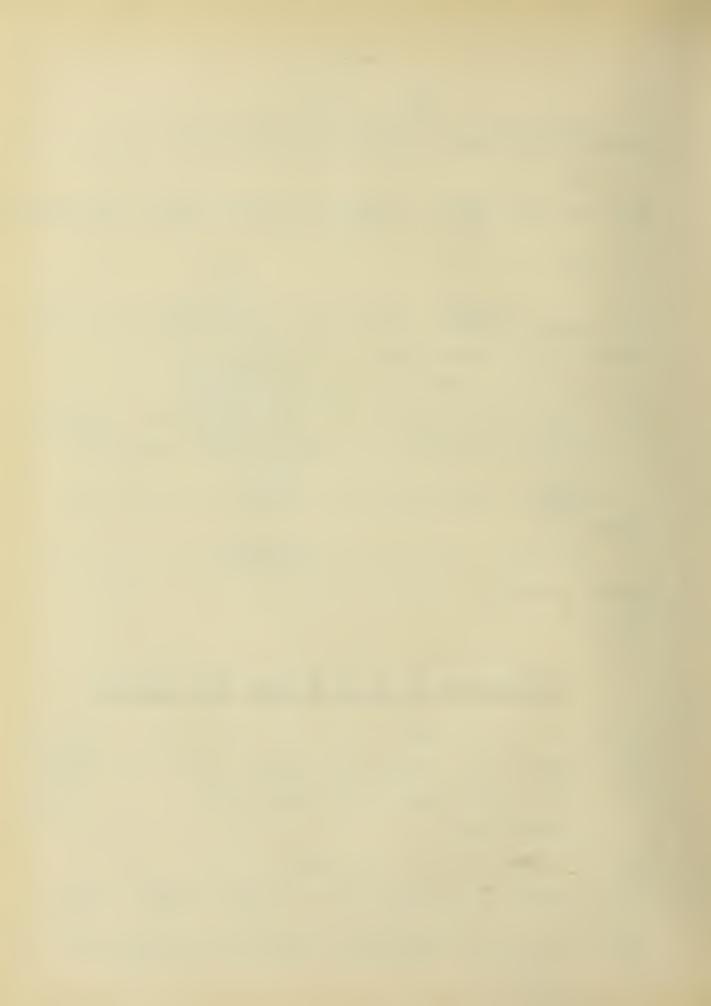
 M_d = 31280 x 20.5 - 39800 x 9.1 - (4510 x 19.05 + 3930 x 16 + 5570 x 12.15 + 4500 x 7.2 + 3620 x 2.2)

= - 900 lb. ft.

 $+ M_1 = +40600$ lb. ft. (see page 6.)

 $- M_1 = 3000 \times 20.5 - 10680 \times 9.7 = 61500 - 103500 = 42000$ lb.ft

But the arch has the same depth at a point 18.3 feet from the



hinge and at this point the negative moment is a maximum so the bending moments there were next computed before further investigating section for a = 20.5.

Investigation of Section 18.3 Feet from Skewback.

$$a = 18.3$$
 $b = 9.1$

 $M_d = 31280 \times 18.3 - 39800 \times 9.1 - (4510 \times 16.85 + 3930 \times 13.8 + 5570 \times 9.95 + 4500 \times 5.04 + 1/2 \times 3620 \times 1.1)$

= + 1000 lb. ft.

 $+M_1 = 6000 \times 18.3 - 73.15 \times (18.3)^2 - 254.5 \times 18.3 \times 9.1$ = +42900 lb. ft.

 $-M_1$ = 3000 x 18.3 - 10680 x 9.1 = -42100 lb. ft. 42900 + 1000 = 43900 lb. ft. maximum moment.

= 527000 lb. in.

The maximum moment therefore occurs at a point 18.3 feet from the skewback and is 527000 lb. in.

The section here is 30 inches deep.

 $C = \frac{M}{1390} = \frac{527000}{1390} = 379 \text{ lb. per sq. in. compression in concrete.}$

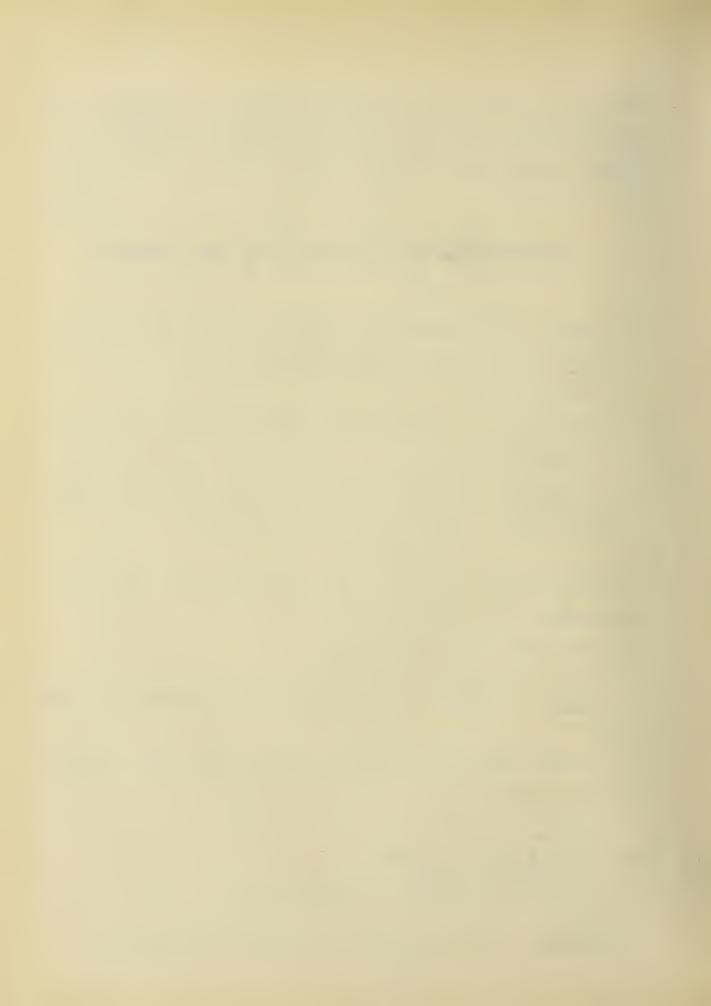
 $v = 31280 + 6000 - 73.15 \times 18.3 - 4510 - 3930 - 5570 - 4500 - 1/2 \times 3620$

= 15600

h = 39800 + 4660 = 44460

 $D = \sqrt{(15600)^2 + (44500)^2} = 47200 \text{ lb.}$

 $\frac{472}{12 \times 30}$ = 131 lb. per sq. in. direct compression.



379 + 131 = 510 lb. per sq. in. maximum compression in concrete.

Investigation of Section 31 Feet from Skewback.

a = 31 b = 11.33

 $M_{\rm d}$ = 31 x 31280 - 11.35 x 39800 - 4510 x 29.55 - 3930 x 26.5 - 5570 x 22.65 - 4500 x 17.7 - 3620 x 12.7 - 5320 x 5.5 = + 1200 lb. ft.

 $+ M_1 = 6000 \times 31 - 73.15 \times (31)^2 - 254.5 \times 31 \times 11.35$ = + 26000 lb. ft.

 $-M_1 = 3000 \times 31 - 10680 \times 11.35 = -28200 lb. ft.$

M = -28200 lb. ft. = -338000 lb. in. maximum moment.

Section 24 inches deep.

 $C = \frac{M}{921} = \frac{338000}{921} = 367 \text{ lb. per sq. in. compression due to}$

bending moment.

h = 39800 + 10680 = 50480 lb.

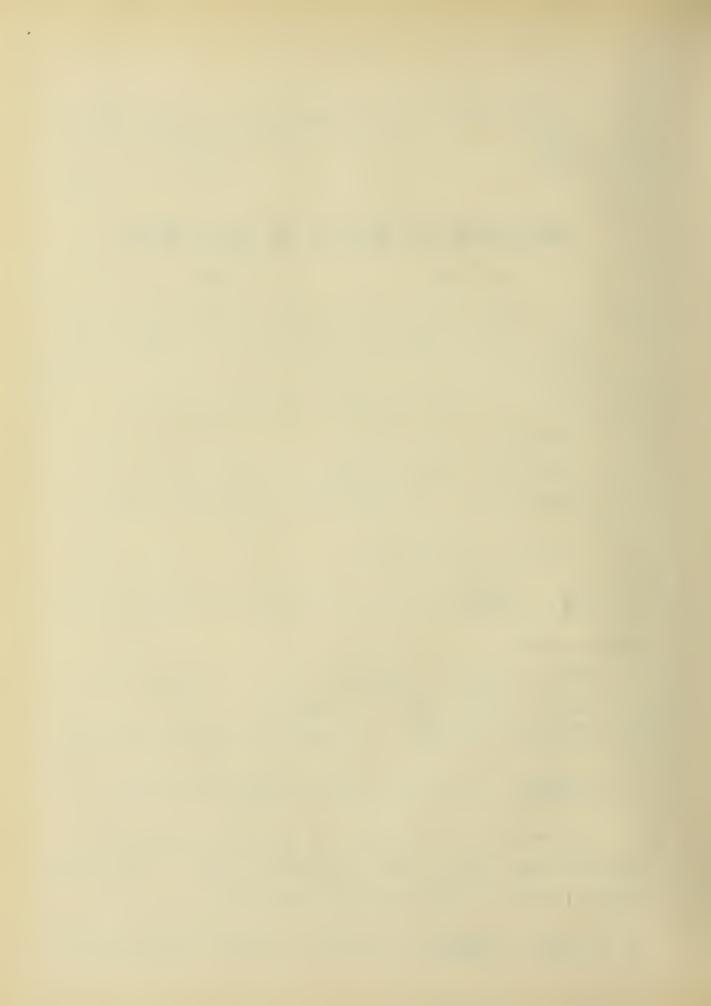
v = 31280 + 3000 - 37450 = 6830 lb.

 $D = \sqrt{(50480)^2 + (6830)^2} = 51000 \text{ lb. direct compression}$

 $\frac{51000}{12 \times 24} = 164$ lb. per sq. in. direct compression

367 + 164 = 531 lb. per sq. in. total compression in concrete, which shows a total compressive stress in the concrete about 6 percent in excess of that permitted.

 $S = \frac{M}{17.7} = \frac{338000}{17.7} = 19100 \text{ lb. per sq. in. tension in}$



steel using the assumption x = 0.26.

But at least $\frac{51000}{2}$ = 25500 lb. tends to compress steel in

tension or $\frac{25500}{.88}$ = 29000 lb. per sq. in. tending to compress the steel in tension, using this assumption. This shows that the neutral axis will not be so far from the tension side as assumed and therefore the compression in the concrete will not be so great as indicated above, so the 6 percent excess may be neglected.

Investigation of Section 35.7 Feet from Skewback.

a = 35.7

b = 11.7

Taking moments of forces to right of section,

 $M_d = 39800 \times 0.1 - 1/2 \times 3830 \times \frac{5.3}{2} = -1090 \text{ lb. ft.}$

 $-M_{\gamma} = 3000 \times 35.7 - 10680 \times 11.7 = -17900 lb. ft.$

 $+M_1 = 6000 \times 35.7 - 73.15 \times (35.7)^2 - 254.5 \times 35.7 \times 11.7$ = -15800 lb. ft.

M = -17900 - 1090 = -18990 lb. ft. = 228000 lb. in. The section here is 22 inches deep.

 $C = \frac{M}{814} = \frac{228000}{814} = 280 \text{ lb. per sq. in. compression in con-}$

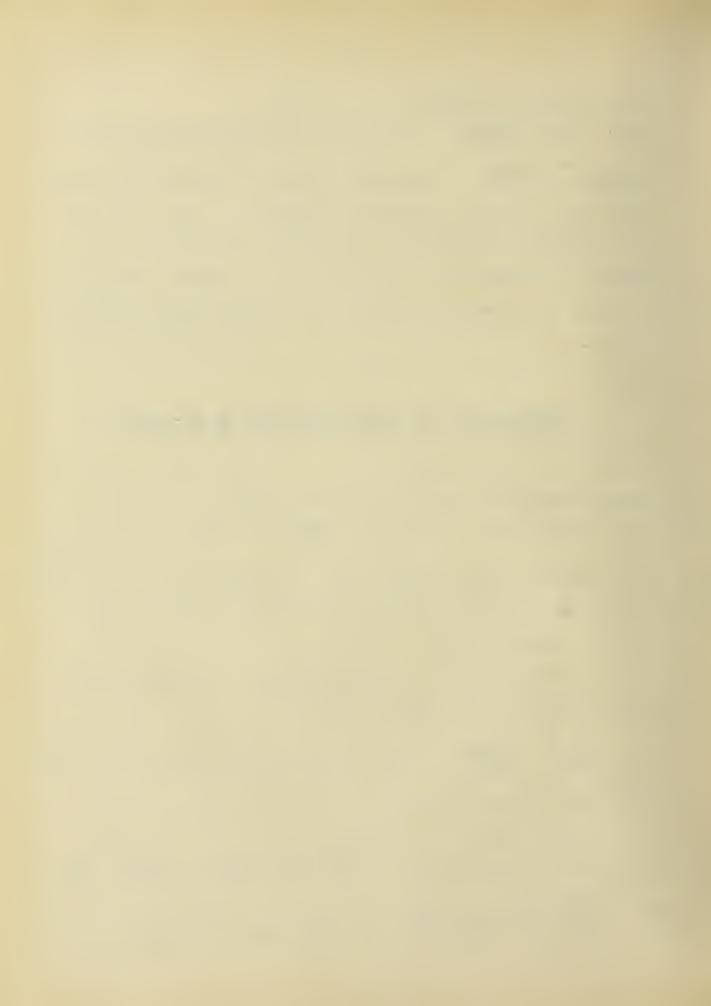
crete due to bending moment.

h = 39800 + 10680 = 50480 lb.

 $v = (v \text{ for section for } a = 31) - 1/2 \times 3830 = 6830 - 1915$

= 49**†**5 lb.

 $D = \sqrt{(4915)^2 + (50480)^2} = 50750 \text{ direct compression.}$



 $\frac{50750}{12 \times 22}$ = 192 lb. per sq. in. direct compression.

280 + 192 = 472 lb. per sq. in. total compression in concrete.

H = 50480 lb.

V = 31280 + 3000 = 34280 lb.

 $R = \sqrt{(50480)^2 + (34280)^2} = 61100 \text{ lb.}$

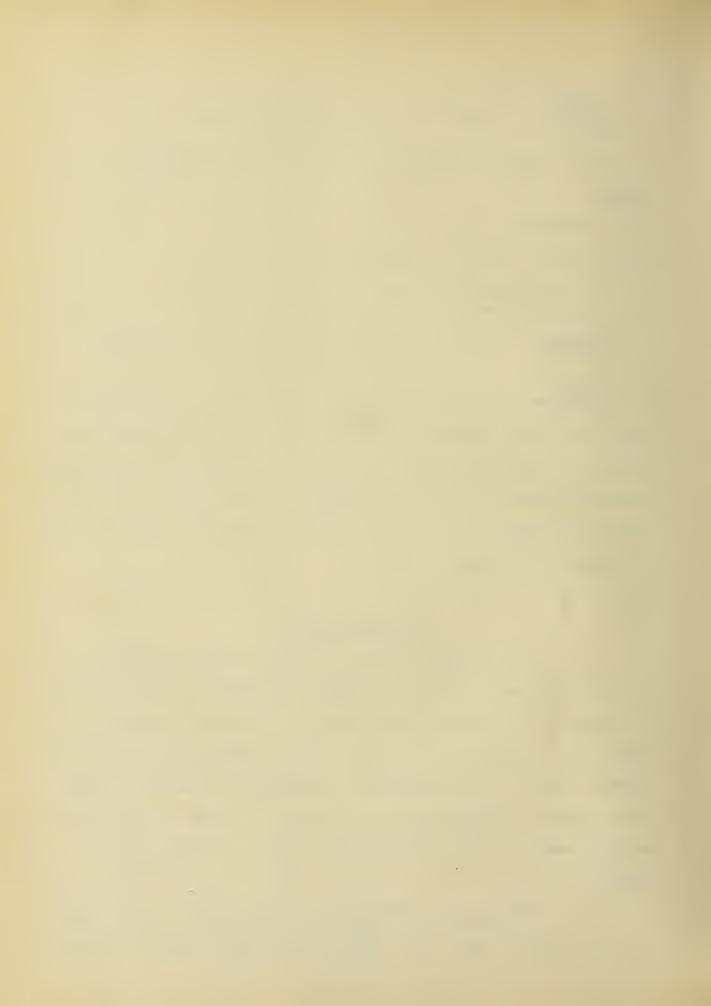
 $\frac{50480}{12 \times 20}$ = 210 lb. per sq. in. under plate at crown.

 $\frac{61100}{12 \times 23}$ = 220 lb. per sq. in. under plate at skewback. This design was therefore accepted. To take the vertical and inclined tensile stresses it was decided to use 3/8" round bars connecting each of the longitudional bars with the one vertically above it. These rods to be bent with a hook on each end and to be spaced 4 feet apart.

SPANDREL WALLS.

If the walls were to be made of plain concrete or masonry the common rule is to make the thickness of any section at least 0.4 the depth of the section. Then for a wall 11 feet deep the thickness should be 4.4 feet at the bottom and about 2 feet at the top. The weight of a section of this wall a foot wide would be $1/2(4.4 + 2) \times 11 \times 150 = 5280$ lb. and the resisting moment about the toe of the base = $5280 \times 2.2 = 11600$ lb. ft.

From Sabine's "Cement and Concrete", p. 402, reinforced concrete beam a foot wide to have this resisting moment



should be about 14 inches deep and have a reinforcement of about 1.8 sq. in. steel. So it was decided to make the wall 14 inches thick at the point where it is 11 feet deep, and 8 inches thick at the top, with 1 1/8 inch round bars placed on the inside and spaced 6 inches apart. On account of the half-rounds to be placed in the form for ornamentation and thus reducing the thickness at that point the total thickness at the bottom of the side walls was made 15 inches.

HINGES.

It was decided to use cast iron hinges as they could be made in any foundry while steel hinges could not. The maximum crown thrust is 39800 + 10680 = 50480 lb. per lineal foot of arch or 4200 lb. per lineal inch.

If this stress were to be on a roller between flat plates the required diameter is determined by $w = 2/3d\sqrt{\frac{2S^3}{E}}$ in which w = load per unit of length, d = diameter of roller, S = the allowable unit stress, E = modulus of elasticity. (See Merriman's "Mechanics of Materials", p. 239).

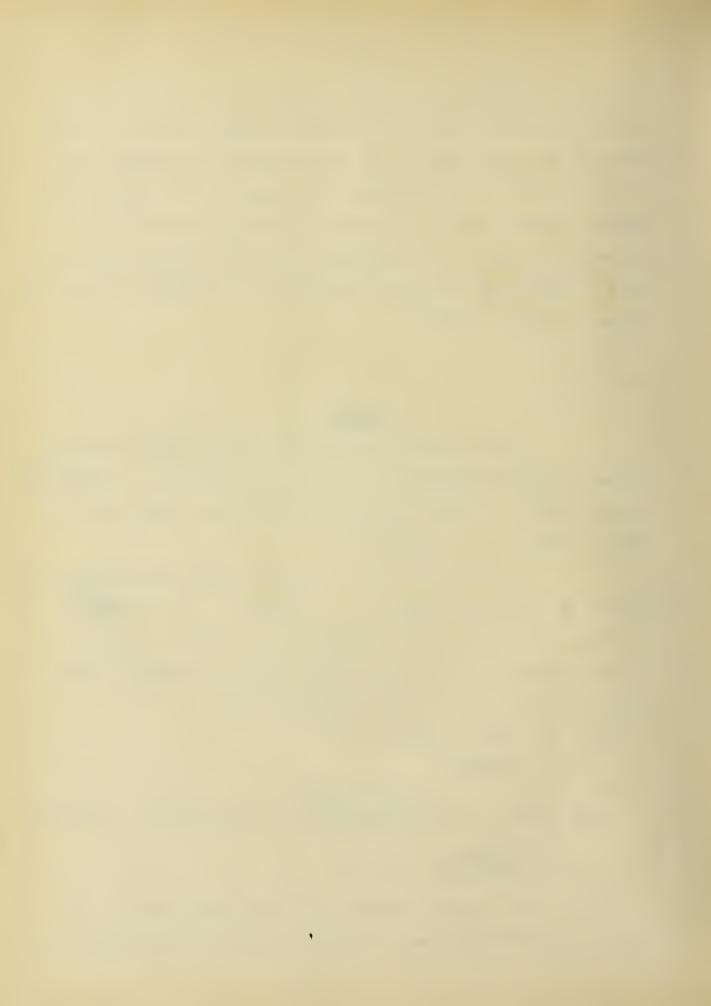
Let S = 6000 lb. per sq. in.

E = 15000000

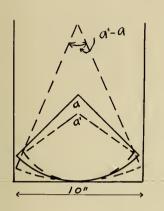
then
$$4200 = 2/3d \sqrt{\frac{2 \times (6000)^3}{150000000}} = 2/3 d \sqrt{28800} = 2/3d \times 170$$
.

$$d = \frac{4200 \times 3}{120 \times 2} = 37. \text{ in.}$$

But suppose a roller of 20 inch radius on a flat plate, and it is required to reduce the radius to 10 inches and



curve the flat surface to keep the relation the same.



Suppose a length of plate of 10 inches be considered and lines be drawn from the extremities of this length perpendicular to the plate. Suppose also a length of 10 inches measured along the circumference of the roller be taken and lines drawn from the ends to the center. The angle a = $\frac{360 \times 10}{2 \text{ N} \times 20}$ =

28.6°. Now bend the plate and increase the curvature of the roller together so that the roller has a radius of 10 inches. Then a' the new angle at the center will equal $\frac{360 \times 10}{2 \text{ T} \times 10}$ or 58° . a' - a = $58 - 29 = 29^{\circ}$ = the increase in the angle between the lines drawn to the plate, that is these lines then make an angle of 29° with each other, or an arc of 29° is 10 inches long.

Circumference = $\frac{360 \times 10}{29}$ = 124 = 2 π R

 $R = \frac{124}{2\pi} = 19.7$ inches or say 19 inches, the radius of the concave face.

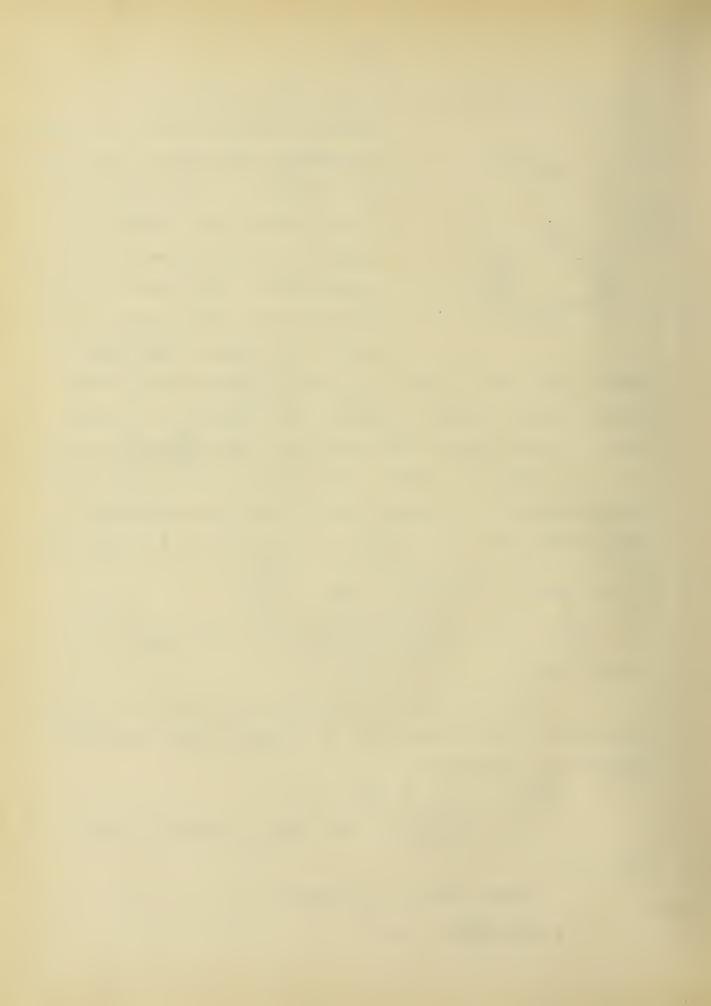
But for greater bearing area this was made 15 inches. The thrust at the skewbacks is, R = 61100 lb. per lineal foot = 5100 lb. per lineal inch.

$$5100 = 2/3 d x 170$$

 $d = \frac{5100 \times 3}{170 \times 2} = 45. \text{ inches diameter of roller on}$ flat plate.

Reduce radius to 15 inches.

$$a = \frac{360 \times 10}{277 \times 45} = 12.7^{\circ}$$



$$a' = \frac{360 \times 10}{2\pi \times 15} = 38^{\circ}$$

$$38^{\circ} - 13^{\circ} = 25^{\circ}$$

$$2 \pi R = \frac{360 \times 10}{25}$$

 $R = \frac{360 \times 10}{25 \times 2\pi} = 23.3 \text{ inches, the radius of the concave face.}$

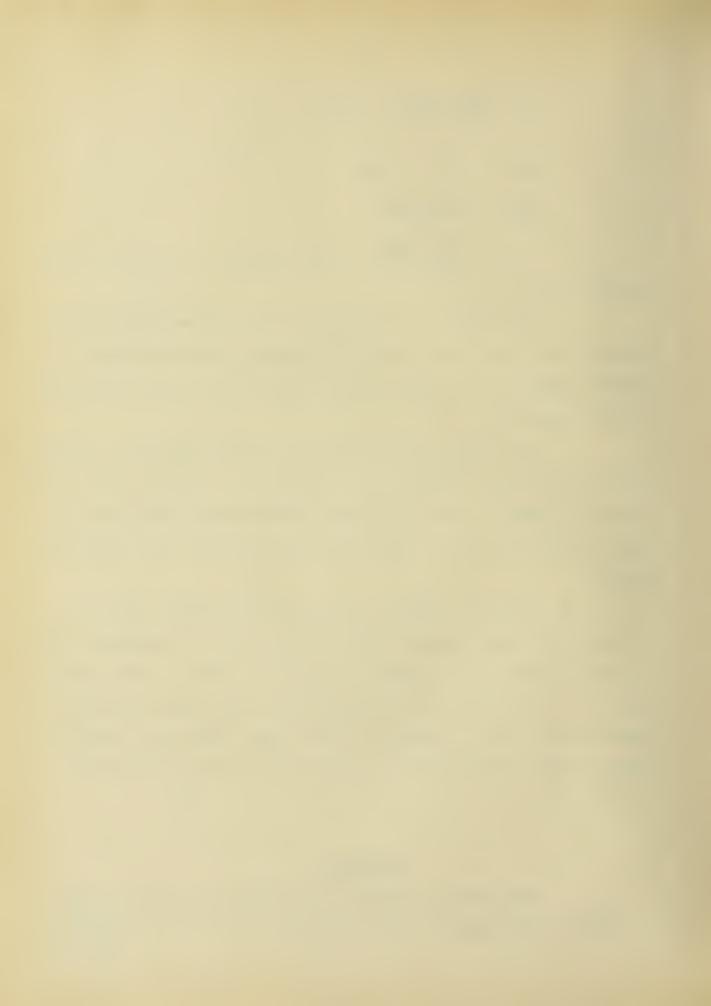
See Plate 4 for details of the hinges which follow somewhat the design for the hinges for the Gruenwald bridge at Munich, Germany, and which are described in the Engineering News of Feb. 23, 1905.

The bolts are 1/2 inch in diameter and are to remain in place until just before the centering is struck. Then these are to be sawed in two and a piece 2 inches long sawed out in such a place as will not interfere with the felt to be placed in later.

After the centers are struck the hinges are to be covered with cement mortar except that a strip of asphalted felt 2 inches thick is to be placed as shown in Plate 5. The hinges are to be cast in three foot lengths and each length will have four bolts to hold the pairs together while erecting. When erected a space of 1/2 inch is to be left between the adjoining pairs.

DRAINAGE.

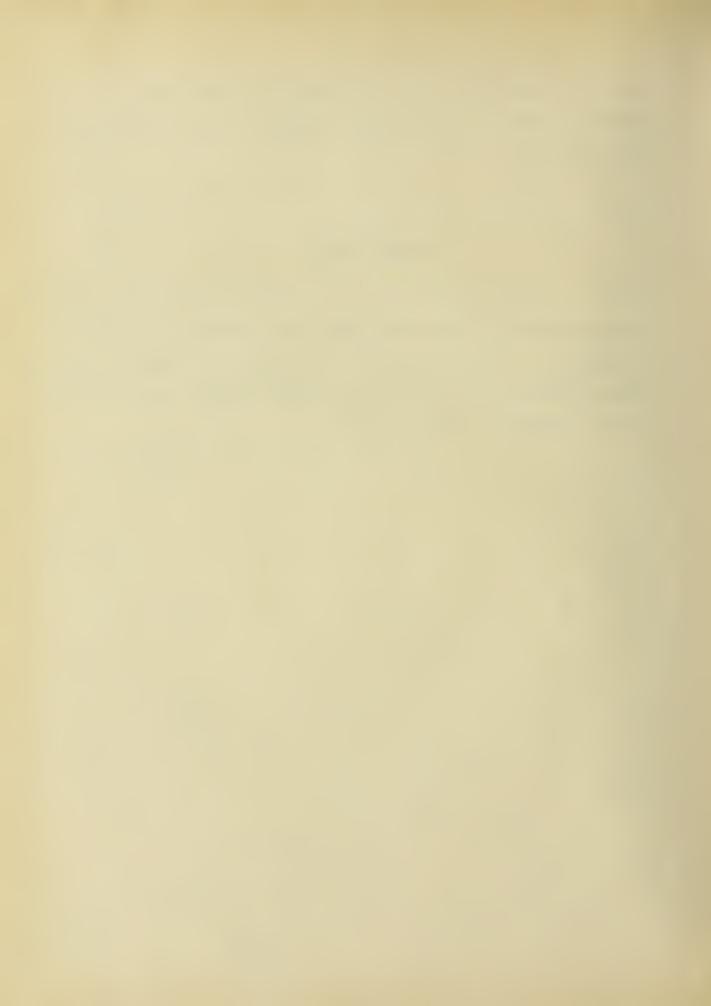
The drainage system is to consist of a row of tile
4 inches inside diameter down the center line of the arch and a



row of the same tile across the arch at the skewback, all to be connected with a 3 inch opening through the arch in the middle. (See Plate 4).

HINGE PLATES.

The crown hinge and the opening at the piers are each to be covered by a two-inch steel plate beveled to an angle of 45 degrees as shown in Plate 4. The bevel is to prevent any possible jamming of dirt or stones between the plate and the sides of the channel in which it rests.



FIRST ARCH. TABLE I.

PRELIMINARY DESIGN.

	WEIGHT			X	WX
SECTION	Earth	Concrete	Total (W)	ft.	lbft.
1	800	2250	3050	36.0	109500
2	3000	2400	5400	26.0	140500
3	1900	1500	3400	18.5	62900
4	2700	1730	4430	13.5	59700
5	3700	1950	5650	8.5	48000
6	2800	1350	4150	4.5	18700
7	3200	1350	455 <u>0</u> 30630	1,5	6800 446100

$$H_{d} = \frac{446100}{11.8} = 37800.$$

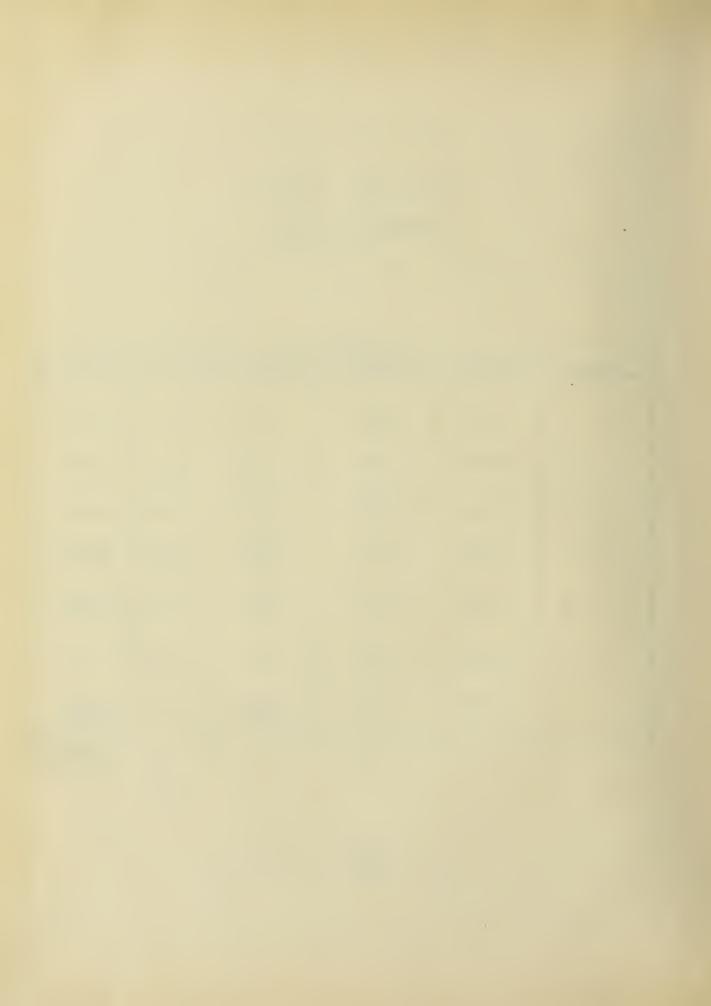


SECOND ARCH. TABLE II.

PRELIMINARY DESIGN.

	WEIGHT			X	WX
SECTION	EARTH	CONCRETE	TOTAL (W)	ft.	lbft.
1	1100	1750	2850	35 . 7	101800
2	1800	2850	4650	25.5	118500
3	1400	2025	3425	18.30	62600
4	2150	2175	4325	13.30	57500
5	3250	2100	5350	8.35	44700
6	2670	1125	3795	4.50	17100
7	3030	900	<u>3930</u> 28325	1.45	5700 407900

$$H_{d} = \frac{107900}{11.8} = 34600.$$

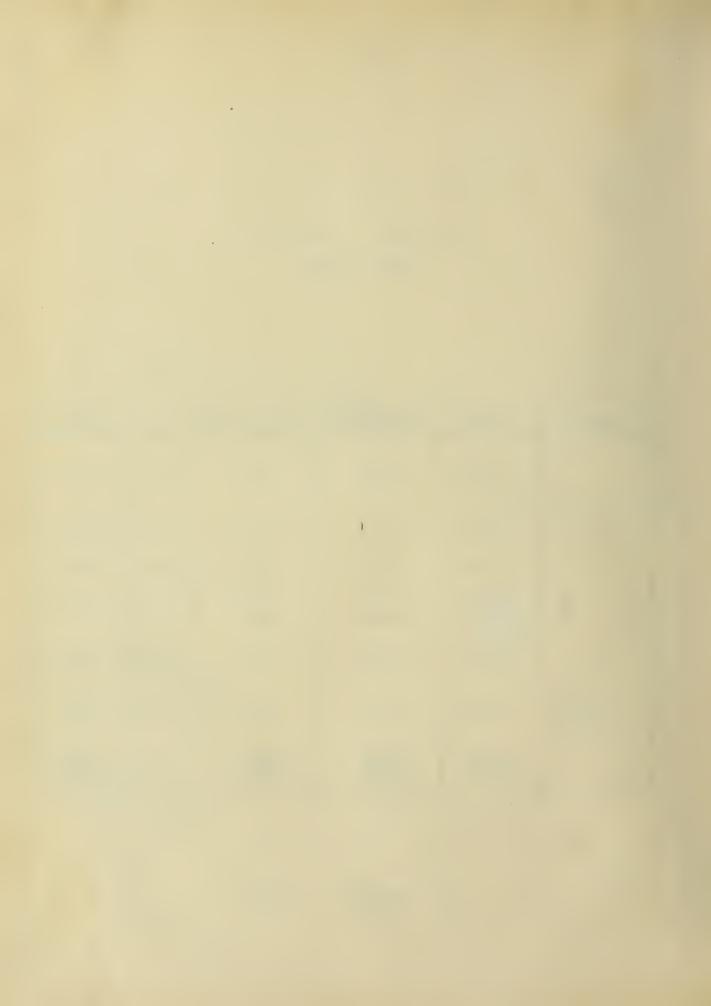


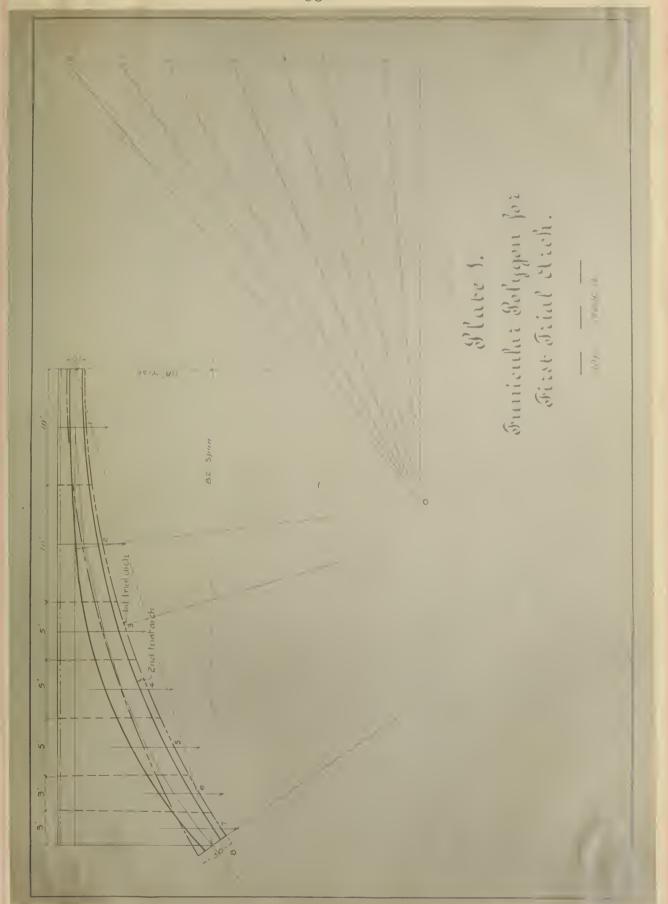
THIRD ARCH. TABLE III.

FINAL DESIGN.

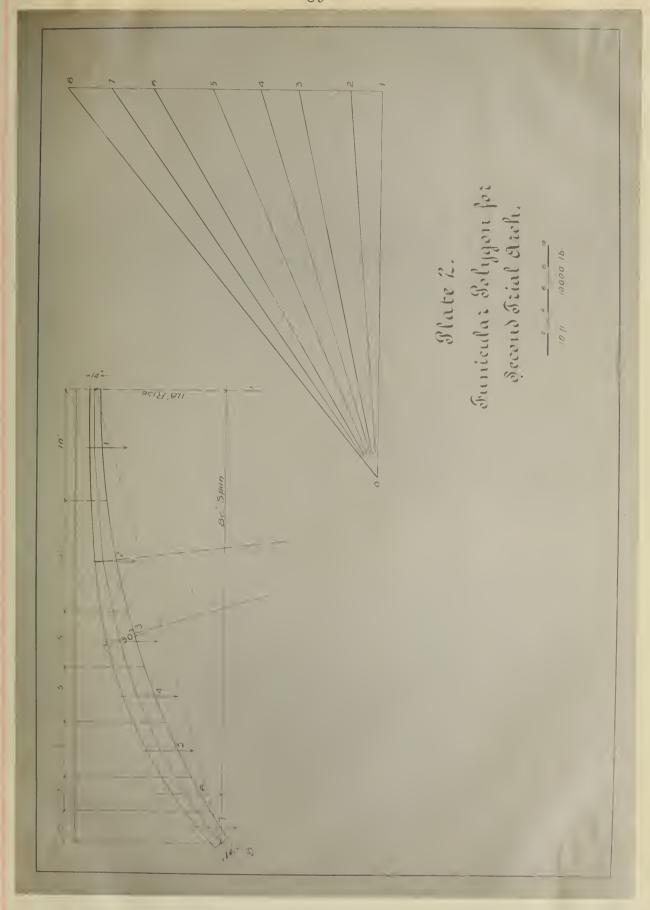
1	1	WEIGHT	X	WX	
SECTION	EARTH	CONCRETE	TOTAL (W)	ft.	lbft
1	1050	2780	3830	35.7	137000
2	1800	3520	5320	25.5	135800
3	1600	2020	3620	18.3	66200
4	2400	2100	4500	13.3	59800
5	3550	2020	5570	8.35	46500
6	2760	1170	3930	4.50	17700
7	3390 16550	1120 14730	4510 31280	1.45	6500 469500

$$H_{d} = \frac{469500}{11.8} = 39800.$$

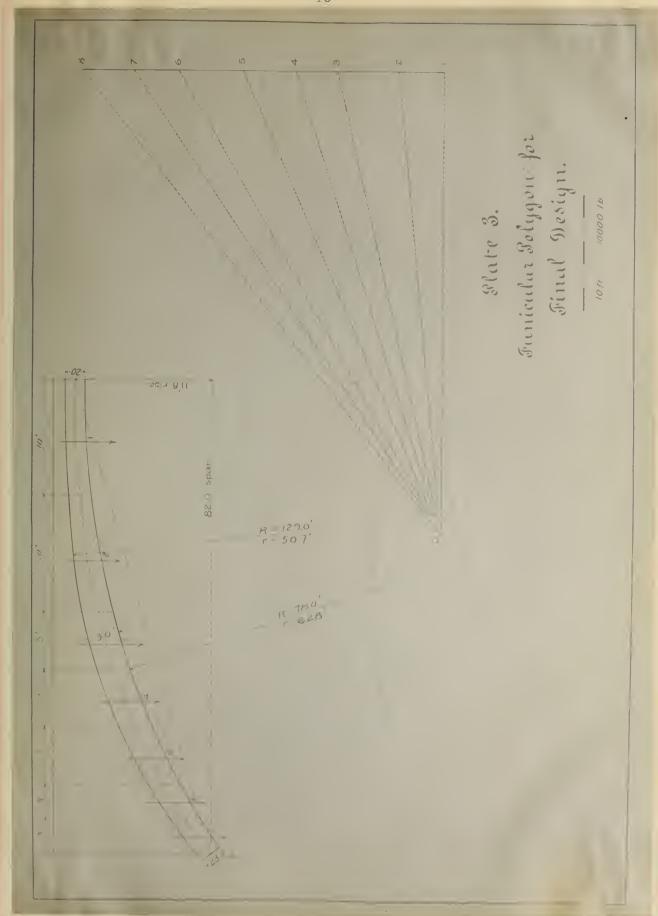




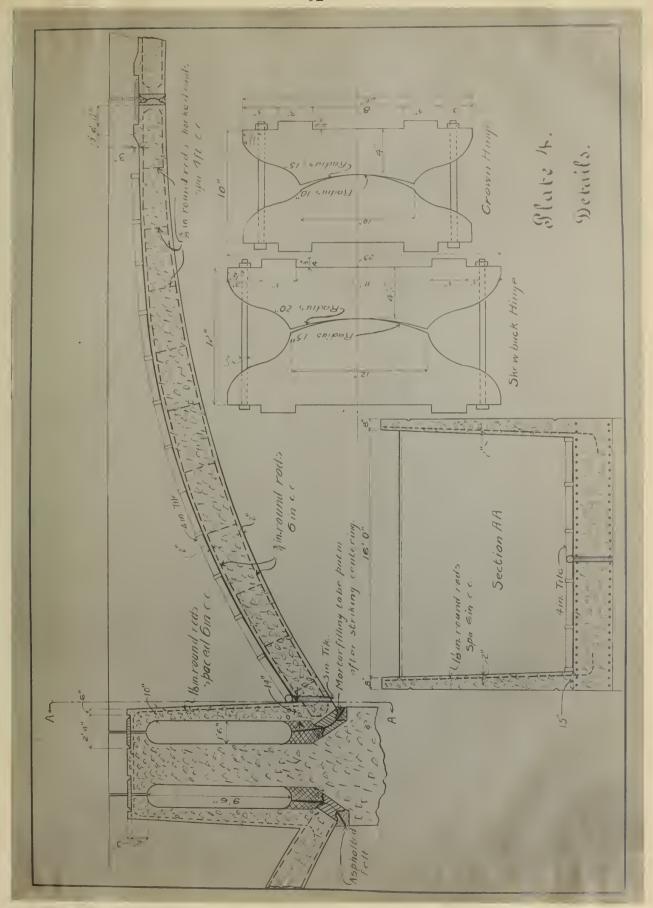
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